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Fuzzy reduced-order filtering for nonlinear parabolic PDE systems with limited communication

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Abstract

This paper investigates a fuzzy reduced-order filter design for a class of nonlinear partial differential equation (PDE) systems. First, a Takagi-Sugeno (T-S) fuzzy model is considered to reconstruct the nonlinear PDE system. Then, the employment of an event-triggered mechanism (ETM) can effectively avoid signal redundancy and improve network resource utilization. Furthermore, based on the advantages of the fuzzy model and ETM, several Lyapunov functions are designed and the proposed filter parameters are obtained by adopting linear matrix inequality methods to satisfy the asymptotic stability condition with H_∞ performance. Finally, a simulation example is presented to demonstrate the practicality and effectiveness of the proposed filter design method.

Keywords: PDE systems, reduced-order fuzzy filtering, event-triggered mechanism, T-S fuzzy model

1. INTRODUCTION

Numerous processes in industry are related not only to time but also to spatial location[1–3], such as nuclear reaction processes, fluid heat exchange processes and biological systems[4–6]. These systems are called distributed parameter systems, usually described by partial differential equations (PDE)[7–9]. According to the different characteristics of the spatial differential operators, the PDE systems can be further divided into three categories, namely, hyperbolic[7], parabolic[8] and elliptic[9]. In particular, parabolic PDEs can be applied to express the dynamic of industrial processes involve diffusion-convection-reaction processes, such as crystal growth processes, semiconductor thermal processes and wavy behavior in chemistry[10,11]. Therefore, control/filtering studies for parabolic PDE systems have attracted extensive attention[12–17]. For example, Wang *et al.*[12] introducted the estimator-based H_∞ sampled-data fuzzy control for nonlinear parabolic PDE systems; Zhang *et al.*[16] addressed the controller design under mobile collocated actuators and sensors; Song *et al.*[17] discussed the reliable H_∞ filter design for PDE system with Markovian jumping sensor faults, which stimulated the author's interest in PDE systems.

In another research field, numerous feasible methods have been developed to solve the analysis and synthesis problem of nonlinear systems. Among them, Takagi-Sugeno (T-S) fuzzy model is widely adopted as an effective method for stability analysis of nonlinear systems[18], which can express any smooth nonlinear function with arbitrary accuracy in any convex compact region[19,20]. In addition, the stability analysis problem of fuzzy control/filtering can be solved by employing the linear matrix inequality (LMI) methods[21]. Based on these advantages of the fuzzy model, many studies have been conducted to apply fuzzy models in controller/filter design for parabolic PDE systems[22,23]. Kerschbaum *et al.*[22] addressed the backstepping control for parabolic PDE systems; Qiu *et al.*[23] addressed the distributed adaptive output feedback consensus problem for parabolic PDE systems.

Meanwhile, the above study mainly used the traditional time-triggered (periodical sampling) network transmission method. The periodic sampling will lead to waste of the network resources and redundancy of transmission signals[24]. Therefore, to better save communication resources, Tabuada[25] proposed an event-triggered mechanism (ETM) that can effectively save network transmission resources, which can determine whether to transmit data according to different trigger conditions[26–32]. For example, Wang *et al.*[26] developed an output-feedback backstepping control method for PDE systems with ETM; based on ETM, Ji *et al.*[27] considered the filtering control for PDE systems. These results prove that, compared to the traditional periodic sampling, ETM can effectively reduce the burden of network bandwidth and improve resource utilization.

Many nonlinear systems are multi-input and multi-output systems[33–37], which the system models are often complex and diverse, how to design simple controllers/filters to meet the corresponding needs. Recently, Su *et al.*[38] proposed a reduced-order filter (ROF) design method, namely, the order of the ROF is lower than the original plant, which will be more favorable for the real-time filtering procedure because some redundancy and extra calculation can be effectively avoided by this filter[39,40]. However, as far as the author knows, there is little work on the design of fuzzy ROF for PDE systems, which arouses the author's interest.

Based on the above discussion, this article intends to design a fuzzy ROF with H_∞ performance for the PDE systems, which its main contributions are as follows:

(1) Different from the fuzzy ROF design[38], the influence of space position is fully considered in the system model and filter design process, the considerations are more comprehensive. Meanwhile, the ROF is designed on the basis of the early work[17], which makes the complex engineering mathematical model more simplified and flexible.

(2) Compared with time-triggered fuzzy filter design[41], the ETM is applied to determine whether to send a sampled signal, which can effectively solve the problem of signal resource transmission in the network.

Organization: Section 2 provides a problem statement and gives relevant definitions and lemmas, which include fuzzy system models, ROF structures and ETM. Section 3 is the main result of this paper, which include stability analysis and ROF design. A numerical example to prove the practicability of the obtained results in Section 4. Section 5 gives conclusions and future research directions.

Notations: $\mathbb{R}^n$ represents n-dimensional Euclidean space; $\mathbb{H}^n$ indicates Hilbert space; The notation N^T and N^{-1} stand for the transpose and inverse matrix of the matrix N, respectively; $X > 0$ (≥ 0) means the matrix X is positive-definite (semi-definite); for ease of expression, define $\psi = \psi(x,t)$, $\varphi = \varphi(x,t)$, $\omega = \omega(x,t)$, $\varphi_o = \varphi_o(x,t)$, $\varphi_m = \varphi_m(x,t)$, $\psi_t = \frac{\partial\psi(x,t)}{\partial t}$, $\psi_{xx} = \frac{\partial\psi(x,t)}{\partial x^2}$, $\theta = \theta(x,t)$, $h_i = h_i(\theta(x,t))$, $\hat{\psi} = \hat{\psi}(x,t)$, $\hat{\varphi}_m = \hat{\psi}_m(x,t)$, $\hat{\varphi}_o = \hat{\psi}_o(x,t)$, $\hat{\psi}_t = \frac{\partial\hat{\psi}(x,t)}{\partial t}$, $\hat{\psi}_{xx} = \frac{\partial\hat{\psi}(x,t)}{\partial x^2}$, $\varphi_m(t_{knT}) = \varphi_m(x, t_kT{+}nT)$, $\varphi_m(t_{kT}) = \varphi_m(x, t_kT)$, $e(i_{kT}) = e_k(x, i_kT)$, $\varphi_{mh} = \varphi_m(x, t-h(t))$, $e_h = e_k(x, t-h(t))$, $\eta_h = \eta(x, t-h(t))$, $\eta_{th} = \eta_(x, t-h)$, $\eta_\theta = \eta(x,\theta)$, $\eta(s) = \eta(x,s)$, $\eta_s(s) = \frac{\partial\eta(x,s)}{\partial s}$, $\xi = \xi(x,t)$; Matrices not explicitly stated in the text are assumed to have the appropriate dimensionality.

2. PROBLEM STATEMENT

2.1. Fuzzy system model

Consider nonlinear distributed parameter systems which are described by PDE as follows:

$$\begin{cases} \psi_t = \Theta\psi_{xx} + f(\psi) + c_1(\psi)\omega, \\ \varphi_o = d(\psi) + c_2(\psi)\omega, \\ \varphi_m = e(\psi), \end{cases} \tag{1}$$

where $\psi \in \mathbb{H}^n$ denotes the state; $x \in [0,l] \subset \mathbb{R}$ and $t > 0$ stand for the space and time, respectively; φ_o represents estimated output signal; φ_m is the measured output; $\omega \in \mathbb{R}^n$ is the considered external disturbance. Θ is a constant matrix. $f(\psi)$, $c_1(\psi)$, $d(\psi)$, $c_2(\psi)$ and $e(\psi)$ are sufficiently smooth nonlinear functions, which satisfy $f(0) = 0$, $c_1(0) = 0$, $d(0) = 0$, $c_2(0) = 0$ and $e(0) = 0$. In this paper, system (1) satisfies the following boundary conditions:

$$\psi_x(0,t) = \psi_x(l,t) = 0 \text{ and } \psi(0,t) = \psi(l,t) = 0. \tag{2}$$

To deal with the nonlinear functions in the systems, the following fuzzy rule is adopted:
Plant Rule $\mathfrak{R}^i$: IF θ_1 is F_1^i ... and θ_z is F_z^i, THEN

$$\begin{cases} \psi_t = \Theta\psi_{xx} + A_i\psi + C_{1i}\omega, \\ \varphi_o = D_i\psi + C_{2i}\omega, \\ \varphi_m = E_i\psi, \end{cases} \tag{3}$$

where $\theta = [\theta_1, \theta_2, \cdots, \theta_z]$ represents the premise variable vector, F_q^i is the fuzzy sets, $i \in \{1, 2, \cdots, r\}$, $q \in \{1, 2, \cdots, z\}$. A_i, C_{1i}, D_i, C_{2i} and E_i are known matrices. Thus, the overall fuzzy system (3) can be expressed as follows:

$$\begin{cases} \psi_t = \Theta\psi_{xx} + \sum\limits_{i=1}^{r} h_i[A_i\psi + C_{1i}\omega], \\ \varphi_o = \sum\limits_{i=1}^{r} h_i[D_i\psi + C_{2i}\omega], \\ \varphi_m = \sum\limits_{i=1}^{r} h_i E_i\psi, \end{cases} \tag{4}$$

where $h_i = \frac{\prod_{q=1}^{z} F_q^i(\theta_q)}{\sum_{i=1}^{r}\prod_{q=1}^{z} F_q^i(\theta_q)}$ with $h_i \geq 0$ and $\sum_{i=1}^{r} h_i = 1$.

2.2. Structure of reduced-order fuzzy filter

Consider the T-S fuzzy model, the following fuzzy ROF is obtained:
Filter Rule $\Re^i$: IF θ_1 is F_1^i ... and θ_z is F_z^i, THEN

$$\begin{cases} \hat{\psi}_t = \Theta_k \hat{\psi}_{xx} + A_{ki}\hat{\psi} + B_{ki}\hat{\varphi}_m, \\ \hat{\varphi}_o = C_{ki}\hat{\psi}, \end{cases} \tag{5}$$

where $\hat{\psi} \in \mathbb{R}^l$ stands for the filter state with $l < n$; the practical filter input signal is $\hat{\varphi}_m$; $\hat{\varphi}_o$ expresses the estimate signal of φ_o; A_{ki}, B_{ki} and C_{ki} are the coefficient matrix of the filter with suitable dimensions. Thus, the overall T-S fuzzy filter can be expressed as follows:

$$\begin{cases} \hat{\psi}_t = \Theta_k \hat{\psi}_{xx} + \sum\limits_{i=1}^{r} h_i[A_{ki}\hat{\psi} + B_{ki}\hat{\varphi}_m], \\ \hat{\varphi}_o = \sum\limits_{i=1}^{r} h_i C_{ki}\hat{\psi}. \end{cases} \tag{6}$$

2.3. Event-triggered mechanism

To improve the utilization of communication resources in the network, this paper introduces samplers and zero-order holder (ZOH) in the network. Consider an ETM, which meets the threshold condition as follows:

$$\int_0^l [\varphi_m(t_{knT}) - \varphi_m(t_{kT})]^T \Omega [\varphi_m(t_{knT}) - \varphi_m(t_{kT})] \leq \varepsilon \int_0^l \varphi_m^T(t_{kT})\Omega\varphi_m(t_{kT}), \tag{7}$$

where $\varepsilon \in [0, 1)$, Ω is a symmetric positive definite matrix, $\varphi_m(t_{knT})$ and $\varphi_m(t_{kT})$ denote the current sampling signal (CSS) and latest transmitted signal (LTS), respectively.

Definition 1. The transmission error $e(i_{kT})$ between the CSS and the LTS as:

$$e(i_{kT}) = \varphi_m(t_{knT}) - \varphi_m(t_{kT}), \tag{8}$$

where $i_{kT} = t_k T + nT$, which implies that the sampling time. Therefore, the transmission error can be expressed as:

$$e(i_{kT}) = \varphi_m(i_{kT}) - \varphi_m(t_{kT}). \tag{9}$$

Based on the ETM, the system (4) can be transformed into a new system with time-delay. This network delay defined as $h(t) = t - i_k T,\ 0 \leq h(t) \leq h$, where $t \in [t_k T + \tau_{tk}, t_{k+1}T + \tau_{tk+1})$. Therefore, the ETM inferred as:

$$\int_0^l e^T(i_{kT})\Omega e(i_{kT})dx \leq \varepsilon \int_0^l \varphi_m^T(t_{kT})\Omega\varphi_m(t_{kT}). \tag{10}$$

By taking into account the influence of the ZOH, the filter input expressed as:

$$\hat{\varphi}_m = \varphi_m(t_{kT}) = \varphi_m(i_{kT}) - e(i_{kT}) = \varphi_{mh} - e_h. \tag{11}$$

Substituting (11) into (6)

$$\begin{cases} \hat{\psi}_t = \Theta_k \hat{\psi}_{xx} + \sum\limits_{i=1}^{r} h_i[A_{ki}\hat{\psi} + B_{ki}\mathcal{H}^T(\varphi_{mh} - e_h)], \\ \hat{\varphi}_o = \sum\limits_{i=1}^{r} h_i C_{ki}\hat{\psi}. \end{cases} \tag{12}$$

Therefore, the above-mentioned ETM is used to convert the fuzzy filter in (6) into a time-delay system.
Remark 1. It is worth noting that the ETM will be reduced to time-triggered communication Mechanism (TTCM) when $\varepsilon \equiv 0$, which means that ETM is more practical in engineering applications.

2.4. Problem formulation

Combining (4) and (12), the corresponding error system is as follows:

$$\begin{cases} \eta_t = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j [\bar{\Theta}\eta + \bar{A}_{ij}\,\eta + \bar{C}_i\omega + \bar{B}_j^{(1)}\eta_h - \bar{B}_j^{(2)} e_h], \\ \tilde{\varphi}_o = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j [\bar{D}_{ij}\eta + C_{2i}\omega], \end{cases} \tag{13}$$

where $\eta \triangleq col\,[\psi\ \hat{\psi}]$, $\tilde{\varphi}_o = \varphi_o - \hat{\varphi}_o$.

$$\bar{\Theta} = \begin{bmatrix} \Theta & 0 \\ 0 & \Theta_k \end{bmatrix}, \bar{A}_{ij} = \begin{bmatrix} A_i & 0 \\ 0 & A_{kj} \end{bmatrix}, \bar{B}_j^{(1)} = \begin{bmatrix} 0 & 0 \\ B_{kj}\mathcal{H}^T E_i & 0 \end{bmatrix},$$
$$\bar{B}_j^{(2)} = \begin{bmatrix} 0 \\ B_{kj}\mathcal{H}^T \end{bmatrix}, \bar{C}_i = \begin{bmatrix} C_{1i} \\ 0 \end{bmatrix}, \bar{D}_{ij} = \begin{bmatrix} D_i & -\mathcal{H}C_{kj} \end{bmatrix}.$$

Remark 2. Because the order of the ROF is lower than the order of the plant. $\mathcal{H} = \begin{bmatrix} I_{l\times l} & 0_{r\times(n-l)} \end{bmatrix}^T$ is introduced to expand the order of the filter, which will be explained in detail in Theorem 2.

Definition 2. The error system (13) with $\omega \equiv 0$ is considered to be asymptotically stable with boundary conditions (2), if satisfies $\lim_{t\to\infty}\left\{\int_0^t \|\eta_\theta\|_2^2\, d\theta\right\} < \infty$ with $\|\eta\|_2 = (\int_0^l \eta^T\eta dx)^{\frac{1}{2}}$.

Definition 3. System (13) is considered to be asymptotically stable with $H\infty$ disturbance attention performance, if there exists a scalar $\gamma > 0$ such that following inequality holds:

$$\int_0^l \int_0^\infty \gamma^{-1}\tilde{\varphi}_o^T\tilde{\varphi}_o dtdx < \int_0^l \int_0^\infty \gamma\omega^T\omega dtdx.$$

Lemma 1.[42] *(Jensens inequality)* Let $y \in \mathbb{H}_l^n$ be a vector function. Then, for any matrix $M > 0$, the following inequality holds:

$$\int_a^b y^T(x)My(x)dx \geq \frac{1}{b-a}\left(\int_a^b y^T(x)dx\right)M\left(\int_a^b y(x)dx\right).$$

Lemma 2.[40] Let $\mu_1, \mu_2, \cdots, \mu_N : \mathbb{R}^m \mapsto \mathbb{R}$ have positive values in the open subset $\mathbb{C}$ of $\mathbb{R}$. Then, the mutually convex combination of μ_i over $\mathbb{C}$ satisfies

$$\min_{\left\{\beta_i|\beta_i>0.\sum_i\beta_i=1\right\}} \sum_i \frac{1}{\beta_i}\mu_i(t) = \sum_i \mu_i(t) + \max_{\varsigma_{i.j}(t)} \sum_{i\neq j} \varsigma_{i.j}(t),$$

Subject to

$$\left\{\varsigma_{i.j}(t) : \mathbb{R}^m \mapsto \mathbb{R}, \varsigma_{j.i}(t) \triangleq \varsigma_{i.j}(t), \begin{bmatrix} \mu_i(t) & \varsigma_{i.j}(t) \\ \varsigma_{i.j}(t) & \mu_j(t) \end{bmatrix} \geq 0\right\}.$$

Consider the following main problem: for the known systems (1), design a fuzzy ROF such that the error system (13) satisfies the asymptotically stable condition with H_∞ performance.

3. RESULTS

In this section, the asymptotically stable condition of the error system (13) is obtained in Theorem 1. The ROF parameters will be obtained by the LMI method in Theorem 2.

Theorem 1. For given scalars $\gamma > 0$, $\varepsilon > 0$ and $h > 0$, error system (13) is asymptotically stable with $H\infty$ performance, if there exist matrices $\Omega > 0$, S satisfying $S\bar{\Theta} \geq 0$, $P > 0$, $Q > 0$, $T_1 > 0$ and G satisfy the following inequality for all $i \in \{1, 2, \cdots, r\}$, $j \in \{1, 2, \cdots, r\}$:

$$\begin{bmatrix} T_1 & G \\ G & T_1 \end{bmatrix} \geq 0, \tag{14}$$

$$\frac{2}{r-1}\Psi_{ii} + \Psi_{ij} + \Psi_{ji} < 0, \tag{15}$$

$$\Psi_{ii} < 0, \tag{16}$$

where

$$\Psi_{ij} = \begin{bmatrix} \Psi_{ij}^{(11)} & \Psi_{ij}^{(12)} \\ * & \Psi_i^{(22)} \end{bmatrix},$$

$$\Psi_{ij}^{(11)} = \begin{bmatrix} \Psi^{11} & \Psi_{ij}^{12} & 0 & \Psi^{14} & 0 \\ * & \Psi^{22} & 0 & \Psi^{24} & G^T \\ * & * & \Psi^{33} & 0 & 0 \\ * & * & * & \Psi^{44} & \Psi^{45} \\ * & * & * & * & \Psi^{55} \end{bmatrix}, \Psi_i^{(22)} = \begin{bmatrix} -(1-\varepsilon)\Omega & 0 & 0 \\ * & -\gamma I & C_{2i}^T \\ * & * & -\gamma I \end{bmatrix},$$

$$\Psi_{ij}^{(12)} = \begin{bmatrix} \Psi_{ij}^{16} & \Psi_{ij}^{17} \\ \Psi_i^{26} & 0 \end{bmatrix}, \Psi_{ij}^{16} = \begin{bmatrix} -S\bar{B}_j^{(2)} & S\bar{C}_i \\ -S\bar{B}_j^{(2)} & S\bar{C}_i \end{bmatrix}, \Psi_{ij}^{17} = \begin{bmatrix} 0 \\ \bar{D}_{ij}^T \end{bmatrix}, \Psi_i^{26} = \begin{bmatrix} 0 & 0 \\ -\bar{E}_i^T\Omega & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Psi^{11} = h^2T_1 - S - S^T, \Psi_{ij}^{12} = P^T + S\bar{A}_{ij} - S^T, \Psi^{14} = S\bar{B}_j^{(1)}, \bar{E}_i = [E_i \;\; 0]^T,$$

$$\Psi^{22} = Q - T_1 + S\bar{A}_{ij} + \bar{A}_{ij}^T S^T, \Psi^{24} = T_1 - G^T + S\bar{B}_j^{(1)}, \Psi^{33} = -S\bar{\Theta} - \bar{\Theta}S^T,$$

$$\Psi^{44} = -2T_1 + G^T + G + \varepsilon\bar{E}_i^T\Omega\bar{E}_i, \Psi^{45} = T_1 - G^T, \Psi^{55} = -Q - T_1.$$

Proof. The Lyapunov function is selected as follows

$$V(t) = \sum_{m=1}^{4} V_m(t), \tag{17}$$

where

$$V_1(t) = \int_0^l \eta^T P\eta dx, V_2(t) = \int_0^l \eta_x^T S\bar{\Theta}\eta_x dx,$$

$$V_3(t) = \int_0^l \int_{t-h}^t \eta^T(s)Q\eta(s)dsdx, V_4(t) = h\int_0^l \int_{-h}^0 \int_{t+\theta}^t \eta_s^T(s)T_1\eta_s(s)dsd\theta dx,$$

The derivative of $V(t)$ is as follows:

$$\begin{aligned} &\dot{V}_1(t) = \int_0^l 2\eta^T P\eta_t dx, \dot{V}_2(t) = -\int_0^l 2\eta_t^T S\bar{\Theta}\eta_{xx} dx, \\ &\dot{V}_3(t) = \int_0^l [\eta^T Q\eta - \eta_h^T Q\eta_h]dx, \dot{V}_4(t) = \int_0^l [h^2\eta_t^T T_1\eta_t - h\int_{t-h}^t \eta_s^T(s)T_1\eta_s(s)ds]dx. \end{aligned} \tag{18}$$

According to Lemmas 1 and 2, the following inequality can be obtained

$$
\begin{aligned}
&-h\int_0^l\int_{t-h}^t \eta_s^T(s)T_1\eta_s(s)dsdx\\
&=-h\int_0^l\{\int_{t-h}^{t-h(t)}\eta_s^T(s)T_1\eta_s(s)ds+\int_{t-h(t)}^t\eta_s^T(s)T_1\eta_s(s)ds\}dx\\
&\leq-\int_0^l\{[\int_{t-h}^{t-h(t)}\eta_s^T(s)ds]T_1[\int_{t-h}^{t-h(t)}\eta_s(s)ds]\\
&+[\int_{t-h(t)}^t\eta_s^T(s)ds]T_1[\int_{t-h(t)}^t\eta_s(s)ds]\\
&+2[\int_{t-h}^{t-h(t)}\eta_s^T(s)ds]G[\int_{t-h(t)}^t\eta_s(s)ds]\}dx.
\end{aligned}\tag{19}
$$

Furthermore, from (13), we can get:

$$
0=2\int_0^l\eta_t^TS[-\eta_t+\bar{\Theta}\eta_{xx}+\sum_{i=1}^r\sum_{j=1}^r h_ih_j(\bar{A}_{ij}\eta+\bar{B}_j^{(1)}\eta_h-\bar{B}_j^{(2)}e_h+\bar{C}_i\omega)]dx.\tag{20}
$$

Under the boundary conditions (2), by partial integration we can obtain:

$$
2\int_0^l\eta^TS\bar{\Theta}\eta_{xx}dx=-\int_0^l\eta_x^T(S\bar{\Theta}+\bar{\Theta}S^T)\eta_xdx.\tag{21}
$$

Consider the ETM (10), define:

$$
\Delta(t)=\varepsilon\int_0^l\varphi_m^T(t_{kT})\Omega\varphi_m(t_{kT})dx-\int_0^l e^T(i_{kT})\Omega e(i_{kT})dx>0,\tag{22}
$$

and

$$
\xi\triangleq col\,[\eta_t\ \ \eta\ \ \eta_x\ \ \eta_h\ \ \eta_{th}\ \ \ e_h\ \ \omega]\,,
$$

Combining (18)-(21) and schur complement, we have:

$$
\dot{V}(t)+\Delta(t)+\int_0^l\gamma^{-1}\tilde{\varphi}_o^T\tilde{\varphi}_o-\gamma\omega^T\omega dx\leq\int_0^l\sum_{i=1}^r\sum_{j=1}^r h_ih_j\xi^T\Psi_{ij}\xi dx.\tag{23}
$$

When $\omega(x,t)\equiv 0$ and according to Theorem 1 and Definition 1, with the processing method [17], we can get the error system (13) is asymptotically stable. Under zero initial condition, one can obtain:

$$
V(\infty)+\int_0^l\int_0^\infty\gamma^{-1}\tilde{\varphi}_o^T\tilde{\varphi}_o-\gamma\omega^T\omega dtdx\leq 0,\tag{24}
$$

which indicates that

$$
\int_0^l\int_0^\infty\tilde{\varphi}_o^T\tilde{\varphi}_odtdx<\int_0^l\int_0^\infty\gamma^2\omega^T\omega dtdx,\tag{25}
$$

which completes the proof.□

Next, solving several LMIs to obtain the parameters of the designed ROF, the main results are as follows:
Theorem 2. For given scalars $\varepsilon\in[0,1)$, $\gamma>0$, $h>0$, if there exist real matrices $W>0$, $Q>0$, $R>0$, $\Omega>0$, $S_1>0$, $T_1>0$, $\tilde{A}_{kj}$, $\tilde{B}_{kj}$, $\tilde{C}_{kj}$ satisfy the following matrix inequations for $i\in\{1,2,\cdots,r\}$, $j\in\{1,2,\cdots,r\}$:

$$
\begin{bmatrix}T_1 & G\\ G & T_1\end{bmatrix}\geq 0,\tag{26}
$$

$$
\frac{2}{r-1}\Phi_{ii}+\Phi_{ij}+\Phi_{ji}<0,\tag{27}
$$

$$
\Phi_{ii}<0,\tag{28}
$$

where

$$\Phi_{ij}=\begin{bmatrix}\Phi^1_{ij} & \Phi^2_{ij}\\ * & \Phi^3_i\end{bmatrix}<0,\Phi^1_{ij}=\begin{bmatrix}\Phi^{11} & \Phi^{12}_{ij} & 0 & \Phi^{14}_{ij} & 0\\ * & \Phi^{22}_{ij} & 0 & \Phi^{24}_{ij} & G^T\\ * & * & \Phi^{33} & 0 & 0\\ * & * & * & \Phi^{44} & \Phi^{45}\\ * & * & * & * & \Phi^{55}\end{bmatrix},\Phi^2_{ij}=\begin{bmatrix}\Phi^{16}_{ij} & 0\\ \Phi^{26}_{ij} & \Phi^{27}_{ij}\\ \Phi^{36}_i & 0\end{bmatrix},$$

$$\Phi^{36}_i=\begin{bmatrix}0 & 0\\ -E_i\Omega & 0\\ 0 & 0\end{bmatrix},\Phi^{27}_{ij}=\begin{bmatrix}D^T_i\\ -\tilde{C}^T_{kj}\mathcal{H}^T\end{bmatrix},\Phi^{24}=\begin{bmatrix}T_{11}-G^T_1+\mathcal{H}\tilde{B}_{kj}\mathcal{H}^TE_i & T_{12}-G^T_3\\ T^T_{12}-G^T_2+\tilde{B}_{kj}\mathcal{H}^TE_i & T_{13}-G^T_4\end{bmatrix},$$

$$\Phi^{12}_{ij}=\begin{bmatrix}\Phi^{121} & \Phi^{122}_j\\ \Phi^{123}_i & \Phi^{124}_j\end{bmatrix},\Phi^{11}=\begin{bmatrix}\Phi^{111} & \Phi^{112}\\ * & \Phi^{113}\end{bmatrix},\Phi^{16}_{ij}=\Phi^{26}_{ij}=\begin{bmatrix}-\mathcal{H}\tilde{B}_{kj}\mathcal{H}^T & S_1C_{1i}\\ -\tilde{B}_{kj}\mathcal{H}^T & W^T\mathcal{H}^TC_{1i}\end{bmatrix},$$

$$\Phi^{22}=\begin{bmatrix}Q_1-T_{11}+S_1A_i & Q_2-T_{12}+\mathcal{H}\tilde{A}_{kj}+A^T_i\mathcal{H}W\\ * & Q_3-T_{13}+\tilde{A}_{kj}+A^T_{kj}\end{bmatrix},\Phi^{14}_{ij}=\begin{bmatrix}\mathcal{H}\tilde{B}_{kj}\mathcal{H}^TE_i & 0\\ \tilde{B}_{kj}\mathcal{H}^TE_i & 0\end{bmatrix},$$

$$\Phi^{55}=\Psi^{55}=-Q-T_1,\Phi^{45}=\Psi^{45}=T_1-G^T,\Phi^{123}_i=P_{21}+W^T\mathcal{H}^TA_i-W^T\mathcal{H}^T,$$

$$\Phi^{112}=h^2T_{12}-2\mathcal{H}W,\Phi^{121}=P_{11}+S_1A_i-S_1,\Psi^{44}=-2T_1+G+G^T+\bar{E}^T_i\Omega\bar{E}_i,$$

$$\Phi^{122}_j=P_{12}+\mathcal{H}\tilde{A}_{kj}-\mathcal{H}W,\Phi^{33}=-\left(S\bar{\Theta}+\bar{\Theta}S^T\right),\Phi^{124}_j=P_{22}+\tilde{A}_{kj}-W^T,$$

$$\Phi^{113}=h^2T_{22}-2W^T,\Phi^{111}=h^2T_{11}-S_1-S^T_1,\Phi^3_i=\Psi^{(22)}_i.$$

then the designed ROF (6) can be obtained by the following relation:

$$\begin{bmatrix}A_{kj} & B_{kj}\\ C_{kj} & 0\end{bmatrix}=\begin{bmatrix}W^{-1} & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}\tilde{A}_{kj} & \tilde{B}_{kj}\\ \tilde{C}_{kj} & 0\end{bmatrix}, \tag{29}$$

Proof. If the conditions (14)-(16) in Theorem 1 are satisfied, then the non-singular matrix can be divided into:

$$S\triangleq\begin{bmatrix}S_1 & \mathcal{H}S_2\\ * & S_3\end{bmatrix}, \tag{30}$$

where

$$\mathcal{H}=\begin{bmatrix}I_{l\times l} & 0_{l\times(n-l)}\end{bmatrix}^T,S_1\in\mathbb{R}^{n\times n},S_2\in\mathbb{R}^{l\times l},S_3\in\mathbb{R}^{l\times l},$$

For the purpose of proving that S_2 is non-singular, define:

$$M=S+\sigma N\ (\sigma>0),$$

and

$$N\triangleq\begin{bmatrix}0_{n\times n} & \mathcal{H}\\ * & 0_{l\times l}\end{bmatrix},M\triangleq\begin{bmatrix}M_1 & \mathcal{H}M_2\\ * & M_3\end{bmatrix}, \tag{31}$$

Since $S>0$, it is easily to obtain $M>0$ for $\sigma>0$. Consequently, it is convenient to verify that M_2 is non-singular for arbitrarily $\sigma>0$ and the above expression is feasible with S. In general, it is assumed that S_2 is non-singular subject to M_2. Based on the above discussion the following definitions can be obtained:

$$U\triangleq\begin{bmatrix}I & 0\\ 0 & S^{-1}_3S^T_2\end{bmatrix},V\triangleq S_1,W\triangleq S_2S^{-1}_3S^T_2, \tag{32}$$

and

$$\begin{bmatrix}\tilde{A}_{kj} & \tilde{B}_{kj}\\ \tilde{C}_{kj} & 0\end{bmatrix}=\begin{bmatrix}S_2 & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}A_{kj} & B_{kj}\\ C_{kj} & 0\end{bmatrix}\begin{bmatrix}S^{-1}_3S^T_2 & 0\\ 0 & I\end{bmatrix}, \tag{33}$$

Then, we can get:

$$U^T SU \triangleq \begin{bmatrix} S_1 & \mathcal{H}W \\ W^T\mathcal{H}^T & W^T \end{bmatrix}, U^T S^T U \triangleq \begin{bmatrix} S_1^T & \mathcal{H}W \\ W^T\mathcal{H}^T & W^T \end{bmatrix},$$

$$U^T S\bar{B}_j^{(1)} = \begin{bmatrix} \mathcal{H}\tilde{B}_{kj}\mathcal{H}^T E_i & 0 \\ \tilde{B}_{kj}\mathcal{H}^T E_i & 0 \end{bmatrix}, U^T S\bar{C}_i \triangleq \begin{bmatrix} S_1 C_{1i} \\ W^T\mathcal{H}^T C_{1i} \end{bmatrix}, \tag{34}$$

$$U^T S\bar{B}_j^{(2)} \triangleq \begin{bmatrix} \mathcal{H}\tilde{B}_{kj} \\ \tilde{B}_{kj} \end{bmatrix}, U^T S\bar{A}_{ij}U \triangleq \begin{bmatrix} S_1 A_i & \mathcal{H}\tilde{A}_{kj}S_3^{-1}S_2^T \\ W^T\mathcal{H}^T A_i & \tilde{A}_{kj} \end{bmatrix}.$$

Pre- and post-multiply both sides of (16) with diag$\{U^T \;\; U^T \;\; I_{1\times 6}\}$ and its transpose, respectively. If (32)-(34) is considered, then the inequality (26)-(28) can hold. Therefore, the error system (13) can be guaranteed to be asymptotically stable with $H\infty$ performance. In addition, (33) is equivalent to

$$\begin{bmatrix} A_{kj} & B_{kj} \\ C_{kj} & 0 \end{bmatrix} = \begin{bmatrix} \left(S_2^{-T}S_3\right)^{-1}W^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{A}_{kj} & \tilde{B}_{kj} \\ \tilde{C}_{kj} & 0 \end{bmatrix} \begin{bmatrix} S_2^{-T}S_3 & 0 \\ 0 & I \end{bmatrix}. \tag{35}$$

Thus, the A_{kj}, B_{kj}, C_{kj} in (6) can be obtained by (35). In case of general, let $S_2^{-T}S_3 = I$, we obtain (29), which can be adopted to construct the ROF in (6). This completes the proof.□

Remark 3. It is noted that the matrix $\mathcal{H}$ presented in Theorem 2 plays a pivotal role in the filter design problem because it is used as a reduced-order factor. When $\mathcal{H}$ is a unit matrix, the designed filter is a full-order filter (FOF). The presence of $\mathcal{H}$ allows for efficient conversion between ROF and FOF.

4. SIMULATION EXAMPLE

In this section, to illustrate the effectiveness of the proposed method, the ROF design problem is studied for the FHN equation, which is a widely adopted model of excitable medium fluctuations in chemistry and can be described as follows:

$$\begin{cases} \psi_{1t} = \psi_{1xx} - \psi_1^3 - 1.2\psi_1 - \psi_2 + 1.2\omega_1, \\ \psi_{2t} = \psi_{2xx} - 0.1\psi_2 + 0.8\omega_1, \end{cases} \tag{36}$$

with boundary (2) and initial conditions

$$\begin{cases} \psi_1(s) = 0.4\cos(\pi x), \\ \psi_2(s) = 0.3\cos(\pi x). \end{cases}$$

The following output and measurement signals are given:

$$\varphi_o = D\psi + C_2\omega, \quad \varphi_m = E\psi,$$

where $\psi = col\,[\psi_1 \;\; \psi_2]$, $\omega = col\,[\omega_1 \;\; \omega_2]$, $\omega_1 = 0.01\sin(x)e^{-t}$, $\omega_2 = 0.1\sin(x)e^{-t}$, $D = -0.05I$, $C2 = -0.01I$, $E = I$.

The systems (36) can be represented by the following fuzzy rule[43]:

Plant Rule 1: IF $\xi(\psi_1)$ is "Big", THEN

$$\psi_t = \psi_{xx} + A_1\psi + C_1\omega,$$

Plant Rule 2: IF $\xi(\psi_1)$ is "Small", THEN

$$\psi_t = \psi_{xx} + A_2\psi + C_1\omega,$$

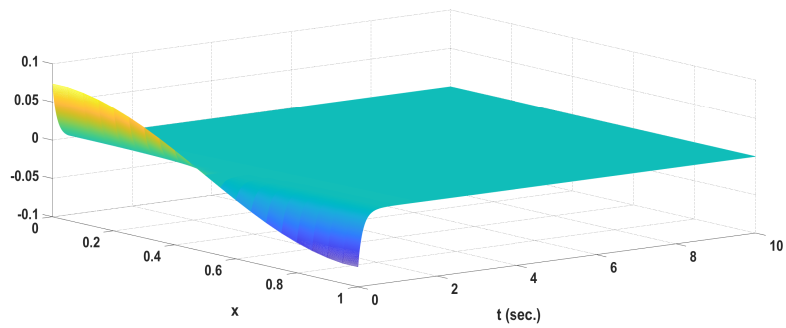

Figure 1. The trajectory of $\tilde{z}_{o1}(x,t)$.

where $\xi(\psi_1) = \psi_1^2$, $A_1 = \begin{bmatrix} -\chi - 1.2 & -1 \\ 0 & -0.1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1.2 & -1 \\ 0 & -0.1 \end{bmatrix}$ and $C_1 = \begin{bmatrix} 1.2 & 0 \\ 0.8 & 0 \end{bmatrix}$ with $\chi \triangleq \max_{\psi_1} \psi_1^2$, $\psi_1 \in [-1.5, 1.5]$, we can get $\chi = 2.55$. The fuzzy membership function can be obtained:

$$h_1(\xi(\psi_1)) = \frac{\xi(\psi_1)}{\chi} = \frac{\psi_1^2}{2.55}, h_2(\xi(\psi_1)) = 1 - h_1(\xi(\psi_1)).$$

Thus, the following T-S fuzzy model is written as follows:

$$\psi_t = \psi_{xx} + \sum_{i=1}^{2} h_1(\xi(\psi_1)) A_i \psi + C_1 \omega,$$

where $A_1 = \begin{bmatrix} -3.45 & -1 \\ 0 & -0.1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1.2 & -1 \\ 0 & -0.1 \end{bmatrix}$. Assume $h = 1ms$, $\varepsilon = 0.05$, $\Theta = 1$, $\Theta_k = 0.5$ and by solving several LMI in Theorem 2, the parameters of the ROF are shown as follows:

$$\Omega = \begin{bmatrix} 1.1539 & -0.9745 \\ -0.6110 & 0.6060 \end{bmatrix}, \; A_{k1} = -3.1938, \; A_{k2} = -1.2967,$$

$$B_{k1} = -0.9575, \; B_{k2} = -1.1876, \; C_{k1} = -0.0510, \; C_{k2} = -0.0550.$$

Finally, to observe the $H\infty$ performance more conveniently, define

$$\xi(t) \triangleq \int_0^t \int_0^1 \left\{ \gamma^{-1} \tilde{\varphi}_o^T(s) \tilde{\varphi}_o(s) - \gamma \omega^T(s) \omega(s) \right\} dxds, \tag{37}$$

Simulation result: the trajectory of the error system (13) is shown in Figures 1 and 2. Figure 3 shows the release moment under the ETM, the trajectory of $\xi(t)$ defined in (37) is shown in Figure 4. It can be observed that the filtering error system is asymptotically stable with $H\infty$ performance. Moreover, the designed ETM (7) can effectively reduce the amount of network signal transmissions and improve the efficiency of network resources utilization.

Remark 4. Inspired by ETM[25], we introduce an ETM in the signal transmission process. Compared with the original TTCM[41], it can effectively reduce the number of signal transmission and improve the network resource utilization. On the other hand, Song *et al.*[17] considered filtering for parabolic PDE systems, but did not consider the problem of ROF design. Compared with Example 2[17], the method proposed in this paper can achieve the same filtering function and the order of the filter is lower than the order of the plant, which simplifies the filter design.

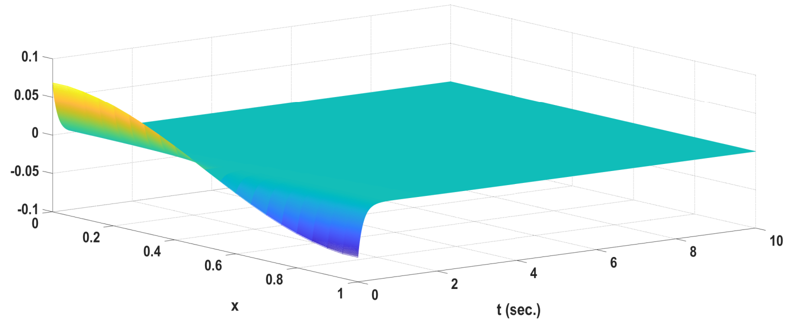

Figure 2. The trajectory of $\tilde{z}_{o2}(x,t)$.

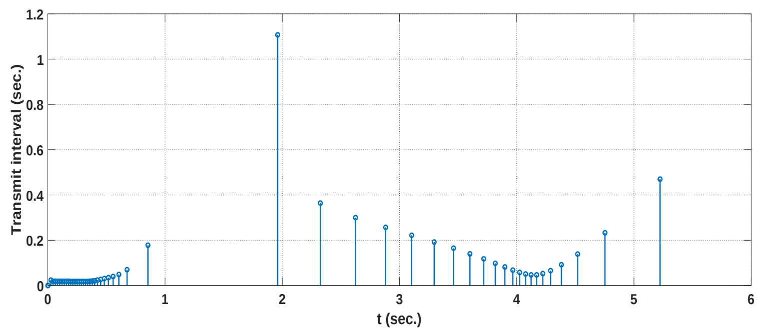

Figure 3. Release instants and release interval.

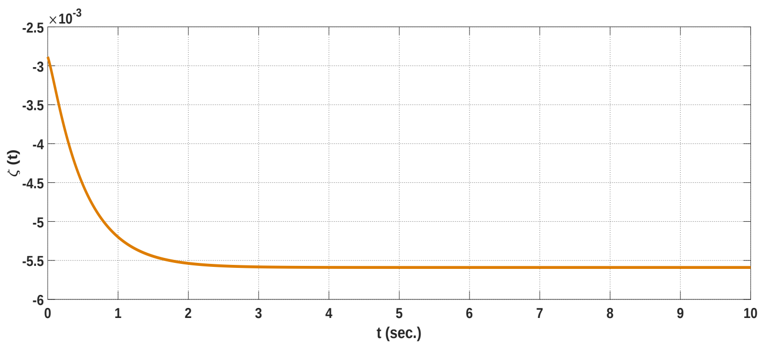

Figure 4. The trajectory of $\xi(t)$ under zero initial condition.

5. CONCLUSIONS

This paper investigates the design method of fuzzy ROF based on ETM for nonlinear parabolic PDE systems. First, a T-S fuzzy model has been considered to reconstruct the nonlinear parabolic PDE systems. In addition, an ETM has been employed to reduce the amount of network transmission data to improve the network resource utilization. Then, the parameters of the designed ROF have been obtained by solving several LMIs based on Lyapunov direct method. Finally, the effectiveness of the proposed method is illustrated by simulation experiments. However, due to the time-delay phenomenon, the systems output signal is difficulty to keep synchronized through the network transmission between the filter and the plant. Therefore, in future studies, we will further consider asynchronous ROF for fuzzy PDE systems, which will be a more interesting topic.

DECLARATIONS

Authors' contributions

Made substantial contributions to experimental studies and manuscript editing: Zhang Z
Made substantial contributions to conception and design of the study: Song X
Made substantial contributions to manuscript revision: Sun X
Made substantial contributions to manuscript preparation: Li C

Availability of data and materials

Not applicable.

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Conflicts of interest

All authors declared that there are no conflicts of interest.

Ethical approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

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