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Decentralized control for interconnected semi-markovian jump systems with partially accessible transition rates: a dynamic memory event-triggered mechanism

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Abstract

This paper investigates the issue of decentralized control for interconnected semi-Markovian systems with partially accessible transition rates (TRs). Firstly, a dynamic system model with a memory event-triggered mechanism (METM) is designed, which can effectively improve the fault tolerance of the event-triggering mechanism by employing the historical trigger data. Then a state feedback control model with dynamic METM is constructed, in which the semi-Markovian parameters with completely unknown and partially known transition probabilities are considered. Some sufficient conditions that insure the stochastic stability of the interconnected semi-Markovian systems can be obtained by utilizing the Lyapunov function and suitable model transformations method. Meanwhile, the parameters and the controller gain matrices of dynamic METM are also solved simultaneously by applying the linear matrix inequalities (LMIs). Finally, a simulation example is given to verify the effectiveness of the proposed method.

Keywords: Decentralized control method, interconnected semi-Markovian jump systems, partially accessible transition rate, dynamic memory event-triggered mechanism.

1. INTRODUCTION

With the prompt development of modern industry, the requirements for system scale and control objectives are increasing. Many single systems improve their own characteristics through interconnection to meet local and global performance requirements[1]. Nowadays, interconnected systems are widely used in actual production and life, such as power systems[2], intelligent transportation systems[3], and network communication systems[4]. The interconnected systems are large-scale composite systems which are composed of several subsystems connected in a specific way. Interconnected systems usually have strong coupling, strong uncertainty, high dimensions, and other characteristics. Thus, the existing traditional control strategies designed for a single system are difficult to directly solve the analysis and control problems of interconnected systems[5,6]. Therefore, many scholars are devoted to the control analysis and design for this kind of large-scale system.

Recently, decentralized control methods have been applied to interconnected control systems, where the subsystem only uses its own information to achieve the control design. Due to its simple structure, low cost, and high reliability, the decentralized control method has drawn wide attention in the control design of large-scale complex systems, and numerous research results have emerged[7,8]. For example, a decentralized control strategy for the linearized power system with different load distributions was studied in[9]. A decentralized adaptive sliding mode control mechanism for the stability of large-scale semi-Markovian jump interconnected systems was proposed in[10,11], and a decentralized output feedback control for large-scale systems with communication delay and random shortcoming measurements was studied in[12]. Recently, it has also witnessed rapid growth in the application of decentralized control methodologies in the field of engineering. For instance, a decentralized Markovian jump H_∞ control routing strategy for mobile multi-agent networked systems were investigated in[13]. The adaptive fuzzy decentralized tracking control for large-scale interconnected nonlinear networked control systems was studied in[14,15], and a Lyapunov-function based event-triggered control was adopted to develop nonlinear discrete-time cyber-physical systems[16]. However, the decentralized control of interconnected systems is still an open field to be developed, and there are still many problems to be discussed.

It is noticed that most actual systems are often affected by some sudden changes during operation, and such systems can be represented by Markovian jump systems (MJSs). However, the residence time in MJSs follows exponential distribution and the distribution of residence time has no memory, that is, the transition rate is a random process independent of past modes, which brings some limitations to its application[17]. In comparison with MJSs, the dwell time of semi-Markovian jump systems (S-MJSs) can obey non-exponential distributions, such as Weibull distribution and Gaussian distribution. The S-MJSs release the limitation of the probability distribution function and reduce the conservatism of the system, thus they have wider application in practice[18]. In recent years, many important theoretical advances and practical significance for S-MJSs can be found. For example, the authors in[19] studied the dynamic output feedback control for a class of linear S-MJSs in the discrete-time domain. The stability of singular switching S-MJSs with uncertain TRs was developed in[20]. In[21], by using the LMI method, the authors studied the stochastic stability of linear S-MJSs, where TRs were divided into different parts. It is often difficult to fully know the jumping probability of modes when the system is modeled as a MJS or S-MJS. It is noticed that the TRs in S-MJSs are more complex because they stick to a more ordinary distribution instead of an exponential distribution[22,23]. Consequently, the study of different forms of TRs would increase the complexity of the process of control design. Recently, an estimation method has been proposed for nonlinear S-MJSs with partially unknown transition probability and output quantization, see[24] and the literature wherein. However, few related works involve exactly unknown and uncertain bounded transition rates of interconnected S-MJSs, which is one of the main motivations of this paper.

Additionally, the event-triggered mechanism (ETM) was drawn to avoid the waste of network resources. Compared with the periodic sampling method, the ETM can avoid the generation of data redundancy[25–27]. However, if the event-triggered threshold argument is a constant, it is difficult to fit in the variety of outside and internal environments. The methods for solving this problem can be summarized as the following two types: the adaptive event-triggered mechanism and the dynamic event-triggered mechanism (DETM)[28,29]. For instance, the authors in[30] proposed an improved dynamic ETM to handle fault detections and isolation issues. And the authors in[31] studied the adaptive event-triggered problem of networked interconnected systems. It should be noted that the majority of triggering conditions are devised based on the diversity between the current sampling signal and the latest released packet[32]. In the above DETM methods, when the relative error between two sampled signals is faint, the current packet is unlikely to be released. Thus, the error message is not sufficient to reflex all dynamic characteristics. A sensitive DETM should consider more system trends in order to achieve a good balance between system performance and utilization of communication resources[33]. For instance, during the transient process, when the system dynamic curve achieves the response peak, the proportional error among two sampled signals is faint[34]. The DETM is unlikely to deliver the packet. However, we expect more sampled signals to be delivered in order to curtail the transient process. For this purpose, we design a weight-based dynamic METM on the base of the existing literature[35,36]. Applying some of the recently released information to ETM has shown to be effective in improving system performance. Obviously, the dynamic METM can appropriately release more packets and get better control performance under the same triggering parameters within a predefined limited time interval[37]. To our knowledge, there are few results on dynamic memory event-triggered control for the interconnected semi-Markovian jump systems, which is the second motivation that lead to our current study.

Enlightened by the viewpoints above, this paper focuses on the decentralized control for a dynamic memory event-triggered interconnected S-MJSs with partially accessible TRs. The main highlights of this paper are summarized below: (1) a decentralized control model for the S-MJSs with partially accessible transition rates is constructed, where a weight-based dynamic METM is first developed to reduce the signal communication burden and save limited broadband resources; (2) construct a semi-Markovian jump mode-depended Lyapunov-Krasovskii functional, and some sufficient conditions are deduced to guarantee the asymptotic stability of the considered system. The controller gain matrices and weighting matrices of dynamic METM are gained in terms of the LMIs technique. Meanwhile, the design scheme proposed is verified via a simulation example.

The rest of this paper is described as below: Interconnected semi-Markovian jump system models with memory event-triggered mechanisms are established in Section 2. Some main results are presented in Section 3. A simulation example is given in Section 4, and a concise conclusion is drawn in Section 5.

Notation: In this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the n-dimensional Euclidean space and the set of $n \times m$ real matrix respectively; the superscripts P^{T} and P^{-1} stand for transposition and inverse, respectively; $diag\{\cdots\}$ indicates a block diagonal matrix; $Q > 0(\geq 0)$ denotes a positive matrix; $\mathbb{E}\{X\}$ submits the mathematical expectation of the stochastic variable X; the notation " $*$ " stands for the symmetric structure.

2. PROBLEM STATEMENT

2.1. System model description

Consider an interconnected semi-Markovian system, which is defined in a fixed probability space (X, F, P) and composed of N subsystems $X_i (i = 1, 2, \cdots, N)$. The dynamic description of the i th subsystem is as follows

$$\dot{x}_i(t) = \mathcal{A}_i(r_t)x_i(t) + \mathcal{B}_i(r_t)u_i(t) + \sum_{j=1, j\neq i}^{N} \mathcal{G}_{ji}(r_t)x_j(t), \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ and $u_i(t)$ represent the state vector of the i th subsystem and control input, respectively. The matrices $\mathcal{A}_i(r_t)$, $\mathcal{B}_i(r_t)$ are of proper dimensions. $\mathcal{G}_{ji}(r_t)$ denotes the interconnection matrix of the i th and j th subsystems; $\{r_t \geq 0\}$ defines a continuous time semi-Markovian process taking discrete values in a finite

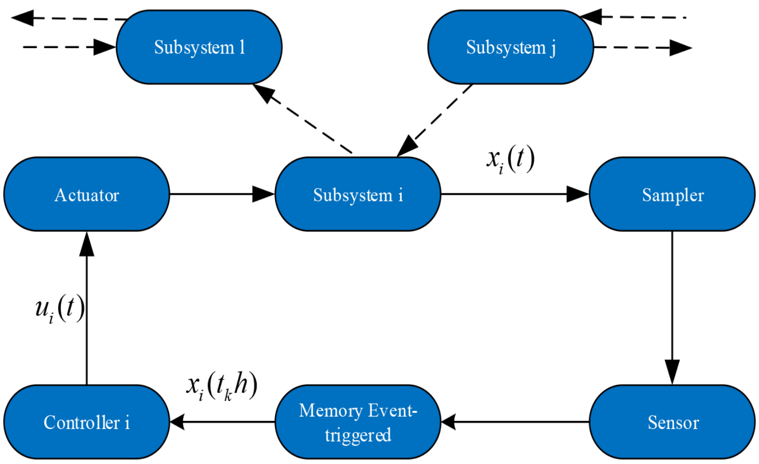

Figure 1. A framework of decentralized control system with METM.

set $S = \{1, 2, \cdots, s\}$ and the generator is given by

$$Pr\{r_{t+\delta} = l | r_t = m\} = \begin{cases} \pi_{ml}(\delta)\delta + o(\delta), m \neq l, \\ 1 + \pi_{mm}(\delta)\delta + o(\delta), m = l, \end{cases} \tag{2}$$

where $\delta > 0$ and $\lim\limits_{\delta \to 0} \frac{o(\delta)}{\delta} = 0$, $\pi_{ml}(\delta) > 0$, $m \neq l$, denotes the transition rate from mode m at time t to mode l at time $t + \delta$, and satisfies $\pi_{mm}(\delta) = -\sum_{l=1, l\neq m}^{s} \pi_{ml}(\delta) < 0$, for each $r_t = m \in S$. More universal uncertain transition rates are taken into account with the following cases. (1) $\pi_{ml}(\delta)$ is completely unknown; (2) $\pi_{ml}(\delta)$ is not completely known but there are upper and lower bounds. In case (2), we assume that $\pi_{ml}(\delta) \in [\underline{\pi}_{ml}, \overline{\pi}_{ml}]$, in which $\underline{\pi}_{ml}$ and $\overline{\pi}_{ml}$ are known real constants meaning the lower and upper bounds of $\pi_{ml}(\delta)$ respectively. The parameter matrix of the system (1) can be abbreviated as $(\mathcal{A}_{im}, \mathcal{G}_{jim}, \mathcal{B}_{im})$. The TRs matrix can be described as

$$\begin{bmatrix} \pi_{11}(\delta) & ? & \pi_{13}(\delta) & \cdots & ? \\ ? & ? & \pi_{23}(\delta) & \cdots & \pi_{2s}(\delta) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ ? & \pi_{s2}(\delta) & ? & \cdots & \pi_{ss}(\delta) \end{bmatrix}$$

where "?" represents a completely unknown element of TRs. For brevity, $\forall m \in S$, let $\bigwedge_m = \bigwedge_{m,k} \bigcup \bigwedge_{m,uk}$, where $\bigwedge_{m,k} = \{l : \pi_{ml}(\delta)$ known upper and lower bounds for $l \in S\}$, $\bigwedge_{m,uk} = \{l : \pi_{ml}(\delta)$ completely unknown for $l \in S\}$.

2.2. Interconnected semi-Markovian jump systems with dynamic METM

To economize network resources and improve data transmission efficiency, here one introduces a dynamic METM. Unlike the other ETM, which only uses instantaneous system information, the proposed METM considers the historically triggered information. Suppose the event-triggered time of the current sampling data is $t_k h$, where $t_k (k = 1, 2, 3, \cdots)$ and h represent some positive integers satisfying $t_k \subset \{0, 1, 2, \cdots\}$ and the sampling period of the sensor, respectively. Define the difference between the latest released sampling data and the current sampling data

$$\Delta_{qi}^{(k)}(t) = x_i(t_{k-q+1}h) - x_i(t_k h + \epsilon h), q = 1, 2, \cdots, M, \tag{3}$$

where $\epsilon \in N_1 = \{1, 2, \cdots\}$, M denotes the memory length, and $t_k h$ indicates the event-triggered instant. The next releasing instant $t_{k+1}h$ is determined as follows

$$t_{k+1}h = t_k h + \min_{\epsilon \in N_1}\{\epsilon h | \sum_{q=1}^{M} \gamma_q (\Delta_{qi}^{(k)}(t))^{\mathrm{T}} \Omega_i \Delta_{qi}^{(k)}(t) > \theta_i(t) x_i^{\mathrm{T}}(t_k h + \epsilon h) \Omega_i x_i(t_k h + \epsilon h)\}, \tag{4}$$

where $\Omega_i > 0$ are the weighting matrix; $\gamma_q \in [0,1]$ are the weighting parameters of the corresponding packet and satisfy $\sum_{q=1}^{M} \gamma_q = 1$. $\theta_i(t)$ are the memory event-triggered threshold and meet the following conditions

$$\dot{\theta}_i(t) = (\frac{1}{\theta_i^2(t)} - \frac{\theta_0}{\theta_i(t)}) \sum_{q=1}^{M} \gamma_q (\Delta_{qi}^{(k)}(t))^{\mathrm{T}} \Omega_i \Delta_{qi}^{(k)}(t), \tag{5}$$

where $\theta_i(t) \in (0,1]$ and $\theta_0 > 0$ is used to regulate the release rate of sampling data. The framework of the decentralized control for interconnected semi-Markovian jump systems with a dynamic METM is shown in Figure 1.

Remark 1. From (5), we can obtain that the dynamic threshold $\theta_i(t)$ is related to the error variable $e_{qi}^{(k)}(t)$. When the error variable tends to zero, for instance, the system tends to be stable at the equilibrium, the dynamic threshold converges to a constant. When $\dot{\theta}_i(t) > 0$, $\theta_i(t)$ is monotonically increasing, which means that the release rate of data at the sampling time will reduce. On the contrary, when $\dot{\theta}_i(t) < 0$, $\theta_i(t)$ is monotonically decreasing, the release rate of data at the sampling time will increase. In particular, when $\dot{\theta}_i(t) \equiv 0$, the event-triggered condition becomes the traditional memory event-triggered condition [22].

Remark 2. By using the historical trigger signals, a memory-base event-triggered condition is proposed in (4), where the past events are assigned appropriate weighting values. This METM can not only save network resources but also can improve the fault tolerance of the event-triggering mechanism compared to the traditional design.

We divide the sampling time interval $[t_k h + \tau_k, t_{k+1} h + \tau_{k+1})$ into $\varepsilon_M + 1$ parts as follows:

$$[t_k h + \tau_k, t_{k+1} h + \tau_{k+1}) = \cup_{l=0}^{\varepsilon_M} I_l, \tag{6}$$

where $l = 0, 1, 2, \cdots, \varepsilon_M, \varepsilon_M = \min\{l | t_k h + (l+1)h + \tau_k \geq t_{k+1} h + \tau_{k+1}\}$ and $I_l = [t_k h + lh + \tau_k, t_{k+1} h + lh + h + \tau_{k+1})$, τ_k denotes the network induced delay. Define delay function $\tau_i(t) = t - (t_k h + \epsilon h)$, and we can get

$$0 \leq \tau_k \leq \tau_i(t) \leq \tau_k + h \leq \tau_M, t \in I_l. \tag{7}$$

Define the error variable $e_{qi}^{(k)}(t) = x_i(t_{k-q+1} h) - x_i(t_k h + \epsilon h)$, and combine the delay function $\tau_i(t) = t - (t_k h + \epsilon h)$, then we can obtain

$$x_i(t_{k-q+1} h) = e_{qi}^{(k)}(t) + x_i(t_k h + \epsilon h) = e_{qi}^{(k)}(t) + x_i(t - \tau_i(t)). \tag{8}$$

The control input $u_i(t)$ in system (1) can be designed as

$$u_i(t) = \sum_{q=1}^{M} K_i^q(r_t) x_i(t_{k-q+1} h) = \sum_{q=1}^{M} K_i^q(r_t) [e_{qi}^{(k)}(t) + x_i(t - \tau_i(t))], t \in I_l. \tag{9}$$

Based on the above analysis, system (1) can be rewritten as

$$\dot{x}_i(t) = \mathcal{A}_{im} x_i(t) + \mathcal{B}_{im} \sum_{q=1}^{M} K_{im}^q x_i(t - \tau_i(t)) + \mathcal{B}_{im} \sum_{q=1}^{M} K_{im}^q e_{qi}^{(k)}(t) + \sum_{j=1, j\neq i}^{N} \mathcal{G}_{jim} x_j(t), \tag{10}$$

where K_{im}^q is the controller gain matrix. Next, a definition and some lemmas will be innovated to deduce the subsequent results of this paper.

Definition 1([16]): Suppose $V(x(t), r_t, t \geq 0)$ is a functional candidate, then the infinitesimal operator $\Im V(t)$ is represented as

$$\Im V(x(t), r_t) = \lim_{\delta \to 0} \frac{E\{V(x(t+\delta), r_{t+\delta}) | x(t), r_t\} - V(x(t), r_t)}{\delta}. \tag{11}$$

Lemma 1([38]): For a given scalar $\mu_i \in (0,1)$, the continuous function $\tau_i(t) \in (0, \tau_M]$ and $\dot{x}_i(t) : [-\tau_M, 0) \rightarrow \mathbb{R}^{n_i}$, there exist positive symmetric matrices $\mathcal{R}_i \in \mathbb{R}^{n_i \times n_i}$ and $\mathcal{S}_i \in \mathbb{R}^{2n_i \times 2n_i}$ to make the inequality hold

$$\tau_M \int_{t-\tau_M}^{t} \dot{x}_i^{\mathrm{T}}(s)\mathcal{R}_i \dot{x}_i(s)ds \geq \omega_1^{\mathrm{T}}\tilde{\mathcal{R}}_{1i}\omega_1 + \omega_2^{\mathrm{T}}\tilde{\mathcal{R}}_{2i}\omega_2 + 2\omega_1^{\mathrm{T}}\mathcal{S}_i\omega_2, \tag{12}$$

where

$$\begin{aligned}
&\tilde{\mathcal{R}}_{1i} = \tilde{\mathcal{R}}_i + (1-\mu_i)(\tilde{\mathcal{R}}_i - \mathcal{S}_i\tilde{\mathcal{R}}_i^{-1}\mathcal{S}_i^{\mathrm{T}}), \tilde{\mathcal{R}}_{2i} = \tilde{\mathcal{R}}_i + \mu_i(\tilde{\mathcal{R}}_i - \mathcal{S}_i^{\mathrm{T}}\tilde{\mathcal{R}}_i^{-1}\mathcal{S}_i), \tilde{\mathcal{R}}_i = diag\{\mathcal{R}_i, 3\mathcal{R}_i\},\\
&\omega_1 = \begin{bmatrix} x_i(t-\tau_i(t)) - x_i(t-\tau_M) \\ x_i(t-\tau_i(t)) + x_i(t-\tau_M) - 2\rho_{1i} \end{bmatrix}, \omega_2 = \begin{bmatrix} x_i(t) - x_i(t-\tau_i(t) \\ x_i(t) + x_i(t-\tau_i(t)) - 2\rho_{2i} \end{bmatrix},\\
&\mathcal{S}_i = \begin{bmatrix} \mathcal{S}_{1i} & \mathcal{S}_{2i} \\ \mathcal{S}_{3i} & \mathcal{S}_{4i} \end{bmatrix}, \rho_{1i} = \frac{1}{\tau_i(t)} \int_{t-\tau_i(t)}^{t} x_i(s)ds, \rho_{2i} = \frac{1}{\tau_M - \tau_i(t)} \int_{t-\tau_M}^{t-\tau_i(t)} x_i(s)ds.
\end{aligned}$$

Lemma 2([39,40]): For a real scalar $\alpha_i > 0$, the matrices $W_i > 0$, $X_{im} > 0$, the following inequality holds

$$-X_{im}W_i^{-1}X_{im} \leq -2\alpha_i X_{im} + \alpha_i^2 W_i, m \in S. \tag{13}$$

3. MAIN RESULTS

Our purpose is to co-design the memory controller (9) and dynamic METM (4) so that system (10) with partially accessible transition rates is stochastically stable. By utilizing the Lyapunov function method, some sufficient conditions that insure the stochastic stability of the interconnected semi-Markovian system (10) are given. Then, a controller design scheme based on LMI is given in Theorem 2.

Theorem 1. For given positive real number $\tau_M > 0$, $\alpha_i > 0$, $\gamma > 0$, $\varepsilon_0 > 0$, $\varepsilon_{ij} > 0$ $(j = 1, 2, 3, \cdots, M+3)$, and $\mu_i \in (0,1)$, the interconnected semi-Markovian jump control system (10) is said to be randomly stable with partially accessible transition rates and dynamic METM if there are positive symmetric matrices $\mathcal{P}_{im} > 0$, $\mathcal{Q}_{im} > 0$, $\mathcal{Q}_i > 0$, $\mathcal{R}_i > 0$, $\Omega_i > 0$, and matrices K_{im}^q, $\mathcal{S}_{1i}$, $\mathcal{S}_{2i}$, $\mathcal{S}_{3i}$ and $\mathcal{S}_{4i}$ with proper dimensions, such that the following matrix inequalities hold:

Case 1. If $\wedge_{m,k} \neq \emptyset$ and $\wedge_{m,uk} \neq \emptyset$, $m \in \wedge_{m,k}$, for $\forall j \in \wedge_{m,uk}$, we have

$$\begin{bmatrix}
\tilde{\Xi}_{im} & * & * & * & * & * & \cdots & * & * & * \\
\Gamma_{im}^{11} & -\check{\mathcal{P}}_{im} & * & * & * & * & \cdots & * & * & * \\
\Gamma_{im}^{21} & 0 & -\mathcal{R}_i & * & * & * & \cdots & * & * & * \\
\Gamma_{im}^{31} & 0 & 0 & -\varepsilon_{i2}\mathcal{R}_i & * & * & \cdots & * & * & * \\
\Gamma_{im}^{41} & 0 & 0 & 0 & -\varepsilon_{i3}\mathcal{R}_i & * & \cdots & * & * & * \\
\Gamma_{im}^{51} & 0 & 0 & 0 & 0 & -\varepsilon_{i4}\mathcal{R}_i & \cdots & * & * & * \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\Gamma_{im}^{61} & 0 & 0 & 0 & 0 & 0 & \cdots & -\varepsilon_{i,M+3}\mathcal{R}_i & * & * \\
\Gamma_{im}^{71} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -\check{\mathcal{R}}_i & * \\
\Gamma_{im}^{81} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\hat{\mathcal{R}}_i
\end{bmatrix} < 0, \tag{14}$$

$$-\mathcal{Q}_i + \sum_{l \in \wedge_{m,k}} \pi_{ml}(h)(\mathcal{Q}_{il} - \mathcal{Q}_{ij}) < 0, \tag{15}$$

where

$$\tilde{\Xi}_{im}=\begin{bmatrix}\tilde{\Xi}_{im}^{11} & * & * & * & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{21} & \tilde{\Xi}_{im}^{22} & * & * & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{31} & \tilde{\Xi}_{im}^{32} & \tilde{\Xi}_{im}^{33} & * & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{41} & \tilde{\Xi}_{im}^{42} & \tilde{\Xi}_{im}^{43} & \tilde{\Xi}_{im}^{44} & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{51} & \tilde{\Xi}_{im}^{52} & -\mathcal{S}_{4i} & \mathcal{S}_{4i} & \tilde{\Xi}_{im}^{55} & * & \cdots & * \\ \tilde{\Xi}_{im}^{61} & 0 & 0 & 0 & 0 & \tilde{\Xi}_{im}^{66} & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\Xi}_{im}^{M+5,1} & 0 & 0 & 0 & 0 & 0 & \cdots & \tilde{\Xi}_{im}^{M+5,M+5}\end{bmatrix},$$

$$\tilde{\Xi}_{im}^{11}=sym\{\mathcal{P}_{im}\mathcal{A}_{im}\}+\tau_M\mathcal{Q}_i-4(1+\mu_i)\mathcal{R}_i+\mathcal{Q}_{im}+\sum_{l\in\wedge_{m,uk}}\frac{\pi_{ml}(h)}{-\lambda_k}[\sum_{l\in\wedge_{m,k}}\pi_{ml}(h)(\mathcal{P}_{il}-\mathcal{P}_{ij})].$$

Case 2. If $\wedge_{m,k}\neq\emptyset$ and $\wedge_{m,uk}\neq\emptyset$, $m\in\wedge_{m,uk}$, for $\forall j\in\wedge_{m,uk}$, we have

$$\begin{bmatrix}\hat{\Xi}_{im} & * & * & * & * & * & \cdots & * & * & * \\ \Gamma_{im}^{11} & -\check{\mathcal{P}}_{im} & * & * & * & * & \cdots & * & * & * \\ \Gamma_{im}^{21} & 0 & -\mathcal{R}_i & * & * & * & \cdots & * & * & * \\ \Gamma_{im}^{31} & 0 & 0 & -\varepsilon_{i2}\mathcal{R}_i & * & * & \cdots & * & * & * \\ \Gamma_{im}^{41} & 0 & 0 & 0 & -\varepsilon_{i3}\mathcal{R}_i & * & \cdots & * & * & * \\ \Gamma_{im}^{51} & 0 & 0 & 0 & 0 & -\varepsilon_{i4}\mathcal{R}_i & \cdots & * & * & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \Gamma_{im}^{61} & 0 & 0 & 0 & 0 & 0 & \cdots & -\varepsilon_{i,M+3}\mathcal{R}_i & * & * \\ \Gamma_{im}^{71} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -\check{\mathcal{R}}_i & * \\ \Gamma_{im}^{81} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\hat{\mathcal{R}}_i\end{bmatrix}<0, \quad (16)$$

$$\mathcal{P}_{im}-\mathcal{P}_{ij}\geq 0, \mathcal{Q}_{im}-\mathcal{Q}_{ij}\geq 0, \quad (17)$$

$$-\mathcal{Q}_i+\sum_{l\in\wedge_{m,k}}\pi_{ml}(h)(\mathcal{Q}_{il}-\mathcal{Q}_{ij})<0, \quad (18)$$

where

$$\hat{\Xi}_{im}=\begin{bmatrix}\hat{\Xi}_{im}^{11} & * & * & * & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{21} & \tilde{\Xi}_{im}^{22} & * & * & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{31} & \tilde{\Xi}_{im}^{32} & \tilde{\Xi}_{im}^{33} & * & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{41} & \tilde{\Xi}_{im}^{42} & \tilde{\Xi}_{im}^{43} & \tilde{\Xi}_{im}^{44} & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{51} & \tilde{\Xi}_{im}^{52} & -\mathcal{S}_{4i} & \mathcal{S}_{4i} & \tilde{\Xi}_{im}^{55} & * & \cdots & * \\ \tilde{\Xi}_{im}^{61} & 0 & 0 & 0 & 0 & \tilde{\Xi}_{im}^{66} & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\Xi}_{im}^{M+5,1} & 0 & 0 & 0 & 0 & 0 & \cdots & \tilde{\Xi}_{im}^{M+5,M+5}\end{bmatrix},$$

$$\begin{aligned}\hat{\Xi}_{im}^{11}&=sym\{\mathcal{P}_{im}\mathcal{A}_{im}\}+\mathcal{Q}_{im}+\tau_M\mathcal{Q}_i-4(1+\mu_i)\mathcal{R}_i\\ &+\sum_{l\in\wedge_{m,uk}}\frac{\pi_{ml}(h)}{-\pi_{mm}(h)-\lambda_k}[\sum_{l\in\wedge_{m,k}}\pi_{ml}(h)(\mathcal{P}_{il}-\mathcal{P}_{ij})+\pi_{mm}(h)(\mathcal{P}_{il}-\mathcal{P}_{ij})].\end{aligned}$$

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Case 3. If $\bigwedge_{m,k} = \emptyset$, $\bigwedge_{m,uk} \neq \emptyset$, $m \in \bigwedge_{m,uk}$, and there exist $l \neq m$ and $l \in \bigwedge_{m,uk}$, we have

$$\begin{bmatrix} \check{\Xi}_{im} & * & * & * & * & * & \cdots & * & * & * \\ \Gamma_{im}^{11} & -\check{\mathcal{P}}_{im} & * & * & * & * & \cdots & * & * & * \\ \Gamma_{im}^{21} & 0 & -\mathcal{R}_i & * & * & * & \cdots & * & * & * \\ \Gamma_{im}^{31} & 0 & 0 & -\varepsilon_{i2}\mathcal{R}_i & * & * & \cdots & * & * & * \\ \Gamma_{im}^{41} & 0 & 0 & 0 & -\varepsilon_{i3}\mathcal{R}_i & * & \cdots & * & * & * \\ \Gamma_{im}^{51} & 0 & 0 & 0 & 0 & -\varepsilon_{i4}\mathcal{R}_i & \cdots & * & * & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \Gamma_{im}^{61} & 0 & 0 & 0 & 0 & 0 & \cdots & -\varepsilon_{i,M+3}\mathcal{R}_i & * & * \\ \Gamma_{im}^{71} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -\check{\mathcal{R}}_i & * \\ \Gamma_{im}^{81} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\hat{\mathcal{R}}_i \end{bmatrix} < 0, \tag{19}$$

where

$$\check{\Xi}_{im} = \begin{bmatrix} \check{\Xi}_{im}^{11} & * & * & * & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{21} & \tilde{\Xi}_{im}^{22} & * & * & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{31} & \tilde{\Xi}_{im}^{32} & \tilde{\Xi}_{im}^{33} & * & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{41} & \tilde{\Xi}_{im}^{42} & \tilde{\Xi}_{im}^{43} & \tilde{\Xi}_{im}^{44} & * & * & \cdots & * \\ \tilde{\Xi}_{im}^{51} & \tilde{\Xi}_{im}^{52} & -\mathcal{S}_{4i} & \mathcal{S}_{4i} & \tilde{\Xi}_{im}^{55} & * & \cdots & * \\ \tilde{\Xi}_{im}^{61} & 0 & 0 & 0 & 0 & \tilde{\Xi}_{im}^{66} & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{\Xi}_{im}^{M+5,1} & 0 & 0 & 0 & 0 & 0 & \cdots & \tilde{\Xi}_{im}^{M+5,M+5} \end{bmatrix},$$

$$\check{\Xi}_{im}^{11} = sym\{\mathcal{P}_{im}\mathcal{A}_{im}\} + \mathcal{Q}_{im} + \tau_M \mathcal{Q}_i - 4(1+\mu_i)\mathcal{R}_i + a_m \pi_{ll}(h)(\mathcal{P}_{im} - \mathcal{P}_{ij}).$$

In addition, the other scalars are given as follows

$$
\begin{aligned}
&\tilde{\Xi}_{im}^{21}=\sum_{q=1}^{M}(K_{im}^{q})^{\mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}\mathcal{P}_{im}-2(1+\mu_i)\mathcal{R}_i-\mathcal{S}_{1i}-\mathcal{S}_{2i}-\mathcal{S}_{3i}-\mathcal{S}_{4i},\\
&\tilde{\Xi}_{im}^{22}=-2(4+\mu_i)\mathcal{R}_i-2\mathcal{S}_{1i}+2\mathcal{S}_{2i}-2\mathcal{S}_{3i}+2\mathcal{S}_{4i}+\Omega_i,\tilde{\Xi}_{im}^{31}=-\mathcal{S}_{1i}-\mathcal{S}_{2i}+\mathcal{S}_{3i}+\mathcal{S}_{4i},\\
&\tilde{\Xi}_{im}^{32}=-2(2-\mu_i)\mathcal{R}_i+\mathcal{S}_{1i}-\mathcal{S}_{2i}-\mathcal{S}_{3i}+\mathcal{S}_{4i},\tilde{\Xi}_{im}^{33}=-\mathcal{Q}_{im}-4(2-\mu_i)\mathcal{R}_i,\\
&\tilde{\Xi}_{im}^{41}=-\mathcal{S}_{3i}-\mathcal{S}_{4i},\tilde{\Xi}_{im}^{42}=\mathcal{S}_{3i}-\mathcal{S}_{4i}+3(2-\mu_i)\mathcal{R}_i,\tilde{\Xi}_{im}^{43}=3(2-\mu_i)\mathcal{R}_i,\tilde{\Xi}_{im}^{44}=-3(2-\mu_i)\mathcal{R}_i,\\
&\tilde{\Xi}_{im}^{51}=3(1+\mu_i)\mathcal{R}_i,\tilde{\Xi}_{im}^{52}=-\mathcal{S}_{2i}+3(1+\mu_i)\mathcal{R}_i,\tilde{\Xi}_{im}^{55}=-3(1+\mu_i)\mathcal{R}_i,\\
&\tilde{\Xi}_{im}^{61}=(K_{im}^{1})^{\mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}\mathcal{P}_{im},\tilde{\Xi}_{im}^{66}=-\theta_0\gamma_1\Omega_i,\tilde{\Xi}_{im}^{M+5,1}=(K_{im}^{M})^{\mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}\mathcal{P}_{im},\tilde{\Xi}_{im}^{M+5,M+5}=-\theta_0\gamma_M\Omega_i,\\
&\Gamma_{im}^{11}=\begin{bmatrix}\mathcal{G}_{ijm}^{\mathrm{T}}\mathcal{P}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & 0\end{bmatrix},\\
&\check{\mathcal{P}}_{im}=-(N-1)^{-1}diag\{\varepsilon_{i1}^{-1}\mathcal{P}_{im},\cdots,\varepsilon_{j1}^{-1}\mathcal{P}_{jm,j\neq i},\cdots,\varepsilon_{N1}^{-1}\mathcal{P}_{Nm}\},\\
&\Gamma_{im}^{21}=\tau_M\mathcal{R}_i\begin{bmatrix}\mathcal{A}_{im} & \mathcal{B}_{im}\sum_{q=1}^{M}K_{im}^{q} & 0 & 0 & 0 & \mathcal{B}_{im}K_{im}^{1} & \cdots & \mathcal{B}_{im}K_{im}^{M}\end{bmatrix},\\
&\Gamma_{im}^{31}=\tau_M\mathcal{R}_i\begin{bmatrix}\mathcal{A}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & 0\end{bmatrix},\\
&\Gamma_{im}^{41}=\tau_M\mathcal{R}_i\begin{bmatrix}0 & \mathcal{B}_{im}\sum_{q=1}^{M}K_{im}^{q} & 0 & 0 & 0 & 0 & \cdots & 0\end{bmatrix},\\
&\Gamma_{im}^{51}=\tau_M\mathcal{R}_i\begin{bmatrix}0 & 0 & 0 & 0 & 0 & \mathcal{B}_{im}K_{im}^{1} & \cdots & 0\end{bmatrix},\\
&\Gamma_{im}^{61}=\tau_M\mathcal{R}_i\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & \cdots & \mathcal{B}_{im}K_{im}^{M}\end{bmatrix},\\
&\Gamma_{im}^{71}=\tau_M\mathcal{R}_i\begin{bmatrix}\mathcal{G}_{ijm} & 0 & 0 & 0 & 0 & 0 & \cdots & 0\end{bmatrix},\\
&\check{\mathcal{R}}_i=-(N-1)^{-1}diag\{(1+\varepsilon_{i2}+\varepsilon_{i3}+\cdots+\varepsilon_{i,M+3})^{-1}\mathcal{R}_1,\cdots,(1+\varepsilon_{j2}+\varepsilon_{j3}+\cdots+\varepsilon_{j,M+3})^{-1}\mathcal{R}_{j,j\neq i},\\
&\cdots,(1+\varepsilon_{N2}+\varepsilon_{N3}+\cdots+\varepsilon_{N,M+3})^{-1}\mathcal{R}_N\},\\
&\Gamma_{im}^{81}=\begin{bmatrix}0 & \mathcal{S}_{1i}+\mathcal{S}_{3i} & -\mathcal{S}_{1i}+\mathcal{S}_{3i} & -\mathcal{S}_{3i} & 0 & 0 & \cdots & 0\\ 0 & \mathcal{S}_{2i}+\mathcal{S}_{4i} & -\mathcal{S}_{2i}+\mathcal{S}_{4i} & -\mathcal{S}_{4i} & 0 & 0 & \cdots & 0\\ \mathcal{S}_{1i}+\mathcal{S}_{3i} & -\mathcal{S}_{1i}+\mathcal{S}_{3i} & 0 & 0 & -\mathcal{S}_{3i} & 0 & \cdots & 0\\ \mathcal{S}_{2i}+\mathcal{S}_{4i} & -\mathcal{S}_{2i}+\mathcal{S}_{4i} & 0 & 0 & -\mathcal{S}_{4i} & 0 & \cdots & 0\end{bmatrix},\\
&\hat{\mathcal{R}}_i=diag\{-(1-\mu_i)^{-1}\mathcal{R}_i,-3(1-\mu_i)^{-1}\mathcal{R}_i,-\mu_i^{-1}\mathcal{R}_i,-3\mu_i^{-1}\mathcal{R}_i\}.
\end{aligned}
$$

Proof: Define the following Lyapunov-Krasovskii functional $V(x(t),r_t)$:

$$
V(x(t),r_t)=\sum_{i=1}^{N}[V_1(x_i(t),r_t)+V_2(x_i(t),r_t)+V_3(x_i(t),r_t)+V_4(x_i(t),r_t)], \tag{20}
$$

where

$$
\begin{aligned}
&V_1(x_i(t),r_t)=x_i^{\mathrm{T}}(t)\mathcal{P}_i(r_t)x_i(t),\\
&V_2(x_i(t),r_t)=\int_{t-\tau_M}^{t}x_i^{\mathrm{T}}(s)\mathcal{Q}_i(r_t)x_i(s)ds+\int_{-\tau_M}^{0}\int_{t+\nu}^{t}x_i^{\mathrm{T}}(s)\mathcal{Q}_ix_i(s)dsd\nu,\\
&V_3(x_i(t),r_t)=\tau_M\int_{-\tau_M}^{0}\int_{t+\nu}^{t}\dot{x}_i^{\mathrm{T}}(s)\mathcal{R}_i\dot{x}_i(s)dsd\nu,\\
&V_4(x_i(t),r_t)=\frac{1}{2}\theta_i^2(t).
\end{aligned}
$$

According to Definition 1, we can get

$$
\begin{aligned}
&\Im V_1(x_i(t), r_t)\\
&= \lim_{\delta\to 0^+}\frac{1}{\delta}\{[\sum_{l=1,l\neq m}^{s} Pr\{r_{t+\delta}=l|r_t=m\}x_i^{\mathrm{T}}(t+\delta)\mathcal{P}_{il}x_i(t+\delta)]\\
&+[Pr\{r_{t+\delta}=m|r_t=m\}x_i^{\mathrm{T}}(t+\delta)\mathcal{P}_{im}x_i(t+\delta)-x_i^{\mathrm{T}}(t)\mathcal{P}_{im}x_i(t)]\}\\
&= \lim_{\delta\to 0^+}\frac{1}{\delta}[\sum_{l=1,l\neq m}^{s}\frac{Pr\{r_{t+\delta}=l, r_t=m\}}{Pr\{r_t=m\}}x_i^{\mathrm{T}}(t+\delta)\mathcal{P}_{il}x_i(t+\delta)]\\
&+\lim_{\delta\to 0^+}\frac{1}{\delta}[\frac{Pr\{r_{t+\delta}=m, r_t=m\}}{Pr\{r_t=m\}}x_i^{\mathrm{T}}(t+\delta)\mathcal{P}_{im}x_i(t+\delta)-x_i^{\mathrm{T}}(t)\mathcal{P}_{im}x_i(t)]\\
&= \lim_{\delta\to 0^+}\frac{1}{\delta}[\sum_{l=1,l\neq m}^{s}\frac{\lambda_{ml}(\phi_m(h+\delta)-\phi_m(h))}{1-\phi_m(h)}x_i^{\mathrm{T}}(t+\delta)\mathcal{P}_{il}x_i(t+\delta)]\\
&+\lim_{\delta\to 0^+}\frac{1}{\delta}[\frac{1-\phi_m(h+\delta)}{1-\phi_m(h)}x_i^{\mathrm{T}}(t+\delta)\mathcal{P}_{im}x_i(t+\delta)-x_i^{\mathrm{T}}(t)\mathcal{P}_{im}x_i(t)]\\
&= \lim_{\delta\to 0^+}\frac{1}{\delta}[\sum_{l=1,l\neq m}^{s}\frac{\lambda_{ml}(\phi_m(h+\delta)-\phi_m(h))}{1-\phi_m(h)}x_i^{\mathrm{T}}(t+\delta)\mathcal{P}_{il}x_i(t+\delta)]\\
&+\lim_{\delta\to 0^+}\frac{1}{\delta}[\frac{1-\phi_m(h+\delta)}{1-\phi_m(h)}(x_i^{\mathrm{T}}(t+\delta)-x_i^{\mathrm{T}}(t))\mathcal{P}_{im}x_i(t+\delta)]\\
&+\lim_{\delta\to 0^+}\frac{1}{\delta}[\frac{1-\phi_m(h+\delta)}{1-\phi_m(h)}x_i^{\mathrm{T}}(t)\mathcal{P}_{im}(x_i(t+\delta)-x_i(t))-\frac{\phi_m(h+\delta)-\phi_m(h)}{1-\phi_m(h)}x_i^{\mathrm{T}}(t)\mathcal{P}_{im}x_i(t)],
\end{aligned}
\tag{21}
$$

where h is the dwell time when the system jumps from the previous mode to mode m, $\phi_m(h)$ represents the cumulative distribution function of residence time when the system (10) maintains in m th mode, λ_{ml} represents the probability density from mode m to mode l. Using the properties of cumulative distribution function, it can be seen that

$$
\lim_{\delta\to 0^+}\frac{1-\phi_m(h+\delta)}{1-\phi_m(h)}=1,\ \lim_{\delta\to 0^+}\frac{1}{\delta}\frac{\phi_m(h+\delta)-\phi_m(h)}{1-\phi_m(h)}=\pi_m(h),
\tag{22}
$$

where $\pi_m(h)$ represents the transition probability of the system in mode m. When $m\neq l$, we have $\pi_{ml}(h)=\lambda_{ml}\pi_m(h)$ and $\pi_{mm}(h)=-\sum_{l=1,l\neq m}^{s}\pi_{ml}(h)$, one derives that $\Im V_1(x_i(t), r_t)=x_i^{\mathrm{T}}(t)(\sum_{l=1,l\neq m}^{s}\pi_{ml}(h)\mathcal{P}_{il})x_i(t)$ $+2x_i^{\mathrm{T}}(t)\mathcal{P}_{im}\dot{x}_i(t)$.

According to [22], there is a scalar $\varepsilon_{i1}^{-1}\in(0,\varepsilon_0^{-1}]$, and we know that

$$
\sum_{i=1}^{N}2x_i^{\mathrm{T}}(t)\mathcal{P}_{im}\sum_{j=1,j\neq i}^{N}\mathcal{G}_{jim}x_j(t)\leq\sum_{i=1}^{N}(\varepsilon_{i1}\sum_{j=1,j\neq i}^{N}x_j^{\mathrm{T}}(t)\mathcal{G}_{jim}^{\mathrm{T}}\mathcal{P}_{im}\sum_{j=1,j\neq i}^{N}\mathcal{G}_{jim}x_j(t)+\varepsilon_{i1}^{-1}x_i^{\mathrm{T}}(t)\mathcal{P}_{im}x_i(t)).
\tag{23}
$$

Similarly, the $\Im V_2(x_i(t), r_t)$, $\Im V_3(x_i(t), r_t)$, $\Im V_4(x_i(t), r_t)$ can be written as

$$
\begin{aligned}
\Im V_2(x_i(t), r_t) &= x_i^{\mathrm{T}}(t)\mathcal{Q}_{im}x_i(t)-x_i^{\mathrm{T}}(t-\tau_M)\mathcal{Q}_{im}x_i(t-\tau_M)\\
&+\tau_M x_i^{\mathrm{T}}(t)\mathcal{Q}_i x_i(t)+\int_{t-\tau_M}^{t}x_i^{\mathrm{T}}(s)(-\mathcal{Q}_i+\sum_{l=1}^{s}\pi_{ml}(h)\mathcal{Q}_{il})x_i(s)ds,\\
\Im V_3(x_i(t), r_t) &= \tau_M^2\dot{x}_i^{\mathrm{T}}(t)\mathcal{R}_i\dot{x}_i(t)-\tau_M\int_{t-\tau_M}^{t}\dot{x}_i^{\mathrm{T}}(s)\mathcal{R}_i\dot{x}_i(s)ds,\\
\Im V_4(x_i(t), r_t) &= \frac{1}{\theta_i(t)}\sum_{q=1}^{M}\gamma_q(\Delta_{qi}^{(k)}(t))^{\mathrm{T}}\Omega_i\Delta_{qi}^{(k)}(t)-\theta_0\sum_{q=1}^{M}\gamma_q(\Delta_{qi}^{(k)}(t))^{\mathrm{T}}\Omega_i\Delta_{qi}^{(k)}(t)\\
&\leq x_i^{\mathrm{T}}(t-\tau_i(t))\Omega_i x_i(t-\tau_i(t))-\theta_0\sum_{q=1}^{M}\gamma_q(e_{qi}^{(k)}(t))^{\mathrm{T}}\Omega_i e_{qi}^{(k)}(t).
\end{aligned}
$$

According to Lemma 1, one further comes to

$$-\tau_M \int_{t-\tau_M}^{t} \dot{x}_i^{\mathrm{T}}(s)\mathcal{R}_i\dot{x}_i(s)ds \leq \xi_i^{\mathrm{T}}(t)\Xi_{im}\xi_i(t) + (1-\mu_i)\omega_1^{\mathrm{T}}\mathcal{S}_i\tilde{\mathcal{R}}_i^{-1}\mathcal{S}_i^{\mathrm{T}}\omega_1 + \mu_i\omega_2^{\mathrm{T}}\mathcal{S}_i^{\mathrm{T}}\tilde{\mathcal{R}}_i^{-1}\mathcal{S}_i\omega_2, \tag{24}$$

where

$$\begin{aligned}
&\xi_i^{\mathrm{T}}(t) = \begin{bmatrix} x_i^{\mathrm{T}}(t) & x_i^{\mathrm{T}}(t-\tau_i(t)) & x_i^{\mathrm{T}}(t-\tau_M) & 2\rho_{1i}^{\mathrm{T}} & 2\rho_{2i}^{\mathrm{T}} \end{bmatrix},\\
&\Xi_{im} = \begin{bmatrix} \Xi_{im}^{11} & * & * & * & * \\ \Xi_{im}^{21} & \Xi_{im}^{22} & * & * & * \\ \Xi_{im}^{31} & \Xi_{im}^{32} & \Xi_{im}^{33} & * & * \\ \Xi_{im}^{41} & \Xi_{im}^{42} & \Xi_{im}^{43} & \Xi_{im}^{44} & * \\ \Xi_{im}^{51} & \Xi_{im}^{52} & \Xi_{im}^{53} & \Xi_{im}^{54} & \Xi_{im}^{55} \end{bmatrix},\\
&\Xi_{im}^{11} = -4(1+\mu_i)\mathcal{R}_i, \Xi_{im}^{21} = -2(1+\mu_i)\mathcal{R}_i - \mathcal{S}_{1i} - \mathcal{S}_{2i} - \mathcal{S}_{3i} - \mathcal{S}_{4i},\\
&\Xi_{im}^{22} = -2(4+\mu_i)\mathcal{R}_i - 2\mathcal{S}_{1i} + 2\mathcal{S}_{2i} - 2\mathcal{S}_{3i} + 2\mathcal{S}_{4i}, \Xi_{im}^{31} = -\mathcal{S}_{1i} - \mathcal{S}_{2i} + \mathcal{S}_{3i} + \mathcal{S}_{4i},\\
&\Xi_{im}^{32} = -2(2-\mu_i)\mathcal{R}_i + \mathcal{S}_{1i} - \mathcal{S}_{2i} - \mathcal{S}_{3i} + \mathcal{S}_{4i}, \Xi_{im}^{33} = -4(2-\mu_i)\mathcal{R}_i, \Xi_{im}^{41} = -\mathcal{S}_{3i} - \mathcal{S}_{4i},\\
&\Xi_{im}^{42} = \mathcal{S}_{3i} - \mathcal{S}_{4i} + 3(2-\mu_i)\mathcal{R}_i, \Xi_{im}^{43} = 3(2-\mu_i)\mathcal{R}_i, \Xi_{im}^{44} = -3(2-\mu_i)\mathcal{R}_i,\\
&\Xi_{im}^{51} = 3(1+\mu_i)\mathcal{R}_i, \Xi_{im}^{52} = -\mathcal{S}_{2i} + 3(1+\mu_i)\mathcal{R}_i, \Xi_{im}^{53} = -\mathcal{S}_{4i}, \Xi_{im}^{54} = \mathcal{S}_{4i}, \Xi_{im}^{55} = -3(1+\mu_i)\mathcal{R}_i.
\end{aligned}$$

And

$$\begin{aligned}
&\dot{x}_i^{\mathrm{T}}(t)\mathcal{R}_i\dot{x}_i(t)\\
&= (\mathcal{A}_{im}x_i(t) + \mathcal{B}_{im}\sum_{q=1}^{M} K_{im}^q x_i(t-\tau_i(t)) + \mathcal{B}_{im}\sum_{q=1}^{M} K_{im}^q e_{qi}^{(k)}(t))^{\mathrm{T}}\\
&\mathcal{R}_i(\mathcal{A}_{im}x_i(t) + \mathcal{B}_{im}\sum_{q=1}^{M} K_{im}^q x_i(t-\tau_i(t)) + \mathcal{B}_{im}\sum_{q=1}^{M} K_{im}^q e_{qi}^{(k)}(t))\\
&+2x_i^{\mathrm{T}}(t)\mathcal{A}_{im}^{\mathrm{T}}\mathcal{R}_i \sum_{j=1,j\neq i}^{N} \mathcal{G}_{jim}x_j(t) + 2x_i^{\mathrm{T}}(t-\tau_i(t))\sum_{q=1}^{M} K_{im}^{q\ \mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}\mathcal{R}_i \sum_{j=1,j\neq i}^{N} \mathcal{G}_{jim}x_j(t)\\
&+2\sum_{q=1}^{M}(e_{qi}^{(k)}(t))^{\mathrm{T}} K_{im}^{q\ \mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}\mathcal{R}_i \sum_{j=1,j\neq i}^{N} \mathcal{G}_{jim}x_j(t) + \sum_{j=1,j\neq i}^{N} x_j^{\mathrm{T}}(t)\mathcal{G}_{jim}^{\mathrm{T}}\mathcal{R}_i \sum_{j=1,j\neq i}^{N} \mathcal{G}_{jim}x_j(t),
\end{aligned}$$

where

$$\sum_{i=1}^{N}(2x_i^{\mathrm{T}}(t)\mathcal{A}_{im}^{\mathrm{T}}\mathcal{R}_i\sum_{j=1,j\neq i}^{N}\mathcal{G}_{jim}x_j(t))$$
$$\leq \sum_{i=1}^{N}((N-1)\sum_{j=1,j\neq i}^{N}\varepsilon_{i2}x_i^{\mathrm{T}}(t)\mathcal{G}_{ijm}^{\mathrm{T}}\mathcal{R}_i\mathcal{G}_{ijm}x_i(t)+\varepsilon_{i2}^{-1}x_i^{\mathrm{T}}(t)\mathcal{A}_{im}^{\mathrm{T}}\mathcal{R}_i\mathcal{A}_{im}x_i(t)),$$
$$\sum_{i=1}^{N}(2x_i^{\mathrm{T}}(t-\tau_i(t))\sum_{q=1}^{M}K_{im}^{q\ \mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}\mathcal{R}_i\sum_{j=1,j\neq i}^{N}\mathcal{G}_{jim}x_j(t))$$
$$\leq \sum_{i=1}^{N}((N-1)\sum_{j=1,j\neq i}^{N}\varepsilon_{i3}x_i^{\mathrm{T}}(t-\tau_i(t))\mathcal{G}_{ijm}^{\mathrm{T}}\mathcal{R}_i\mathcal{G}_{ijm}x_i(t-\tau_i(t))$$
$$+\varepsilon_{i3}^{-1}x_i^{\mathrm{T}}(t-\tau_i(t))\sum_{q=1}^{M}K_{im}^{q\ \mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}\mathcal{R}_i\mathcal{B}_{im}\sum_{q=1}^{M}K_{im}^{q}x_i(t-\tau_i(t)),$$
$$\sum_{i=1}^{N}(2\sum_{q=1}^{M}(e_{qi}^{(k)}(t))^{\mathrm{T}}K_{im}^{q\ \mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}\mathcal{R}_i\sum_{j=1,j\neq i}^{N}\mathcal{G}_{jim}x_j(t))$$
$$\leq \sum_{i=1}^{N}((N-1)\sum_{j=1,j\neq i}^{N}\varepsilon_{i4}x_i^{\mathrm{T}}(t)\mathcal{G}_{ijm}^{\mathrm{T}}\mathcal{R}_i\mathcal{G}_{ijm}x_i(t)+\varepsilon_{i4}^{-1}\sum_{q=1}^{M}(e_{qi}^{(k)}(t))^{\mathrm{T}}K_{im}^{q\ \mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}\mathcal{R}_i\mathcal{B}_{im}\sum_{q=1}^{M}K_{im}^{q}e_{qi}^{(k)}(t)),$$
$$\sum_{i=1}^{N}(\sum_{j=1,j\neq i}^{N}x_j^{\mathrm{T}}(t)\mathcal{G}_{jim}^{\mathrm{T}}\mathcal{R}_i\sum_{j=1,j\neq i}^{N}\mathcal{G}_{jim}x_j(t))\leq\sum_{i=1}^{N}(N-1)\sum_{j=1,j\neq i}^{N}x_i^{\mathrm{T}}(t)\mathcal{G}_{ijm}^{\mathrm{T}}\mathcal{R}_i\mathcal{G}_{ijm}x_i(t).$$

Let $\eta_i(t)=\begin{bmatrix}\xi_i^{\mathrm{T}}(t) & (e_{1i}^{(k)}(t))^{\mathrm{T}} & \cdots & (e_{Mi}^{(k)}(t))^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$, then we obtain

$$\begin{aligned}\dot{V}(x(t),r_t)\leq&\sum_{i=1}^{N}\eta_i^{\mathrm{T}}(t)[\Gamma_{im}^{0}+(\Gamma_{im}^{11})^{\mathrm{T}}\mathcal{P}_{im}^{-1}\Gamma_{im}^{11}+(\Gamma_{im}^{21})^{\mathrm{T}}\mathcal{R}_i^{-1}\Gamma_{im}^{21}+\varepsilon_{i2}^{-1}(\Gamma_{im}^{31})^{\mathrm{T}}\mathcal{R}_i^{-1}\Gamma_{im}^{31}+\varepsilon_{i3}^{-1}(\Gamma_{im}^{41})^{\mathrm{T}}\mathcal{R}_i^{-1}\Gamma_{im}^{41}\\&+\varepsilon_{i4}^{-1}(\Gamma_{im}^{51})^{\mathrm{T}}\mathcal{R}_i^{-1}\Gamma_{im}^{51}+\varepsilon_{i,M+1}^{-1}(\Gamma_{im}^{61})^{\mathrm{T}}\tilde{\mathcal{R}}_i^{-1}\Gamma_{im}^{61}+(\Gamma_{im}^{71})^{\mathrm{T}}\check{\mathcal{R}}_i^{-1}\Gamma_{im}^{71}(\Gamma_{im}^{81})^{\mathrm{T}}\hat{\mathcal{R}}_i^{-1}\Gamma_{im}^{81}]\eta_i(t)\\=&\sum_{i=1}^{N}\eta_i^{\mathrm{T}}(t)\Pi_{im}\eta_i(t),\end{aligned}$$

where $\Gamma_{im}^{0}=\tilde{\Xi}_{im}(case1)$, $\Gamma_{im}^{0}=\hat{\Xi}_{im}(case2)$, $\Gamma_{im}^{0}=\check{\Xi}_{im}(case3)$.

Therefore, if $\Pi_{im}<0$ and $-\mathcal{Q}_i+\sum_{l=1}^{s}\pi_{ml}(h)\mathcal{Q}_{il}<0$, the interconnected semi-Markovian control system (10) with partially accessible TRs and dynamic METM is stochastically stable. Considering the partially accessible transition rates, we will get the corresponding conclusion from the following three cases.

Case 1. If $\bigwedge_{m,k}\neq\emptyset$ and $\bigwedge_{m,uk}\neq\emptyset$, $m\in\bigwedge_{m,k}$, denote $\lambda_k=\sum_{l\in\bigwedge_{m,k}}\pi_{ml}(h)$. Since $\bigwedge_{m,uk}\neq\emptyset$, then one derives that $\lambda_k<0$, thus $\sum_{l=1}^{s}\pi_{ml}(h)\mathcal{P}_{il}$ can be presented as

$$\sum_{l=1}^{s}\pi_{ml}(h)\mathcal{P}_{il}=(\sum_{l\in\bigwedge_{m,k}}+\sum_{l\in\bigwedge_{m,uk}})\pi_{ml}(h)\mathcal{P}_{il}=\sum_{l\in\bigwedge_{m,k}}\pi_{ml}(h)\mathcal{P}_{il}-\lambda_k\sum_{l\in\bigwedge_{m,uk}}\frac{\pi_{ml}(h)}{-\lambda_k}\mathcal{P}_{il}. \tag{25}$$

Obviously, there exists $0\leq\frac{\pi_{ml}(h)}{-\lambda_k}\leq 1(l\in\bigwedge_{m,uk})$ and $\sum_{l\in\bigwedge_{m,uk}}\frac{\pi_{ml}(h)}{-\lambda_k}=1$. So for $\forall j\in\bigwedge_{m,uk}$, there is

$$\sum_{l=1}^{s}\pi_{ml}(h)\mathcal{P}_{il}=\sum_{l\in\bigwedge_{m,uk}}\frac{\pi_{ml}(h)}{-\lambda_k}(\sum_{l\in\bigwedge_{m,k}}\pi_{ml}(h)(\mathcal{P}_{il}-\mathcal{P}_{ij})). \tag{26}$$

According to Schur complement, it is inferred that (14) is equivalent to $\Pi_{im} < 0$ and (15) is equivalent to $-\mathcal{Q}_i + \sum_{l=1}^{s} \pi_{ml}(h)\mathcal{Q}_{il} < 0$.

Case 2. If $\wedge_{m,k} \neq \emptyset$ and $\wedge_{m,uk} \neq \emptyset$, $m \in \wedge_{m,uk}$, denote $\lambda_k = \sum_{l\in\wedge_{m,k}} \pi_{ml}(h)$. Since $\wedge_{m,k} \neq \emptyset$, we know that $\lambda_k > 0$, thus $\sum_{l=1}^{s} \pi_{ml}(h)\mathcal{P}_{il}$ can be presented as

$$\begin{aligned}\sum_{l=1}^{s} \pi_{ml}(h)\mathcal{P}_{il} &= \sum_{l\in\wedge_{m,k}} \pi_{ml}(h)\mathcal{P}_{il} + \pi_{mm}(h)\mathcal{P}_{im} + \sum_{l\in\wedge_{m,uk},l\neq m} \pi_{ml}(h)\mathcal{P}_{il} \\ &= \sum_{l\in\wedge_{m,k}} \pi_{ml}(h)\mathcal{P}_{il} + \pi_{mm}(h)\mathcal{P}_{im} - (\pi_{mm}(h)+\lambda_k) \sum_{l\in\wedge_{m,uk},l\neq m} \frac{\pi_{ml}(h)}{-\pi_{mm}(h)-\lambda_k}\mathcal{P}_{il}.\end{aligned} \tag{27}$$

It is obvious that $0 \leq \frac{\pi_{ml}(h)}{-\pi_{mm}(h)-\lambda_k} \leq 1 (l \in \wedge_{m,uk})$ and $\sum_{l\in\wedge_{m,uk}} \frac{\pi_{ml}(h)}{-\pi_{mm}(h)-\lambda_k} = 1$. So for $\forall j \in \wedge_{m,uk}, j \neq m$, there is

$$\sum_{l=1}^{s} \pi_{ml}(h)\mathcal{P}_{il} = \sum_{l\in\wedge_{m,uk},l\neq m} \frac{\pi_{ml}(h)}{-\pi_{mm}(h)-\lambda_k} (\sum_{l\in\wedge_{m,k}} \pi_{ml}(h)(\mathcal{P}_{il} - \mathcal{P}_{ij}) + \pi_{mm}(h)(\mathcal{P}_{il} - \mathcal{P}_{ij})). \tag{28}$$

By applying Schur complement and $\pi_{mm}(h) = -\sum_{l=1,l\neq m}^{s} \pi_{ml}(h)$, it is deduced that (16), (17) are equivalent to $\Pi_{im} < 0$ and (18) is equivalent to $-\mathcal{Q}_i + \sum_{l=1}^{s} \pi_{ml}(h)\mathcal{Q}_{il} < 0$.

Case 3. If $\wedge_{m,k} = \emptyset$ and $\wedge_{m,uk} \neq \emptyset$, $m \in \wedge_{m,uk}$, assume there exists $l \neq m$ and $l \in \wedge_{m,uk}$. Denote $\lambda_k = \pi_{mm}(h) = a_m\pi_{ll}(h)$. Noting that $\lambda_k < 0$, $\sum_{l=1}^{s} \pi_{ml}(h)\mathcal{P}_{il}$ can be presented as

$$\sum_{l=1}^{s} \pi_{ml}(h)\mathcal{P}_{il} = \pi_{mm}(h)\mathcal{P}_{im} + \sum_{l\in\wedge_{m,uk},l\neq m} \pi_{ml}(h)\mathcal{P}_{il} = \pi_{mm}(h)\mathcal{P}_{im} - \lambda_k \sum_{l\in\wedge_{m,uk},l\neq m} \frac{\pi_{ml}(h)}{-\lambda_k}\mathcal{P}_{il}. \tag{29}$$

It is obvious that $\sum_{l\in\wedge_{m,uk},l\neq m} \pi_{ml}(h) = -\pi_{mm}(h) = -\lambda_k > 0$. So for $\forall j \in \wedge_{m,uk}, j \neq m$, there is

$$\sum_{l=1}^{s} \pi_{ml}(h)\mathcal{P}_{il} = \sum_{l\in\wedge_{m,uk},l\neq m} \frac{\pi_{ml}(h)}{-\lambda_k} [\pi_{mm}(h)(\mathcal{P}_{im} - \mathcal{P}_{ij})] = \pi_{mm}(h)(\mathcal{P}_{im} - \mathcal{P}_{ij}) = a_m\pi_{ll}(h)(\mathcal{P}_{im} - \mathcal{P}_{ij}). \tag{30}$$

By applying Schur complement and $\pi_{mm}(h) = -\sum_{l=1,l\neq m}^{s} \pi_{ml}(h)$, we konw that (19) is equivalent to $\Pi_{im} < 0$.

In summary, if inequalities (14) − (19) hold, the interconnected semi-Markovian control system (10) with partially accessible TRs and dynamic METM is stochastically stable.

Remark 3. Theorem 1 designed sufficient conditions to ensure (10) is stochastically stable. However, it is difficult to directly use this result to acquire the controller gain matrices. Theorem 2 gives a LMI-based sufficient criterion for the solvability.

Theorem 2. For given a positive real number $\tau_M > 0$, $\alpha_i > 0$, $\gamma > 0$, $\varepsilon_0 > 0$, $\varepsilon_{ij} > 0$ $(j = 1, 2, 3, \cdots, M + 3)$, $\mu_i \in (0, 1)$ and $\pi_{ml}(h) \in [\underline{\pi}_{ml}, \overline{\pi}_{ml}]$, the system (10) with partially accessible TRs and dynamic METM is stochastically stable if there are positive symmetric matrices $X_{im} > 0$, $Y_{im}^q > 0$, $\overline{\mathcal{Q}}_{im} > 0$, $\overline{\mathcal{Q}}_i > 0$,$\overline{\mathcal{R}}_i > 0$, $\overline{\Omega}_i > 0$, and matrices K_{im}^q, $\overline{\mathcal{S}}_{1i}$, $\overline{\mathcal{S}}_{2i}$, $\overline{\mathcal{S}}_{3i}$ and $\overline{\mathcal{S}}_{4i}$ with proper dimensions, such that the linear matrix inequalities hold:

Case 1. If $\bigwedge_{m,k} \neq \emptyset$ and $\bigwedge_{m,uk} \neq \emptyset$, $m \in \bigwedge_{m,k}$, for $\forall j \in \bigwedge_{m,uk}$, we have

$$\begin{bmatrix} \overline{\Xi}_{im,\overline{m}} & * & * & * & * & * & \cdots & * & * & * \\ \overline{\Gamma}^{11}_{im} & \overline{\Gamma}^{12}_{im} & * & * & * & * & \cdots & * & * & * \\ \overline{\Gamma}^{21}_{im} & 0 & \Phi(\overline{\mathcal{R}}_i) & * & * & * & \cdots & * & * & * \\ \overline{\Gamma}^{31}_{im} & 0 & 0 & \varepsilon_{i2}\Phi(\overline{\mathcal{R}}_i) & * & * & \cdots & * & * & * \\ \overline{\Gamma}^{41}_{im} & 0 & 0 & 0 & \varepsilon_{i3}\Phi(\overline{\mathcal{R}}_i) & * & \cdots & * & * & * \\ \overline{\Gamma}^{51}_{im} & 0 & 0 & 0 & 0 & \varepsilon_{i4}\Phi(\overline{\mathcal{R}}_i) & \cdots & * & * & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \overline{\Gamma}^{61}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & \varepsilon_{i,M+3}\Phi(\overline{\mathcal{R}}_i) & * & * \\ \overline{\Gamma}^{71}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -\check{\overline{\mathcal{R}}}_i & * \\ \overline{\Gamma}^{81}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\hat{\overline{\mathcal{R}}}_i \end{bmatrix} < 0, \tag{31}$$

$$\begin{bmatrix} \overline{\Xi}_{im,\underline{m}} & * & * & * & * & * & \cdots & * & * & * \\ \overline{\Gamma}^{11}_{im} & \overline{\Gamma}^{12}_{im} & * & * & * & * & \cdots & * & * & * \\ \overline{\Gamma}^{21}_{im} & 0 & \Phi(\overline{\mathcal{R}}_i) & * & * & * & \cdots & * & * & * \\ \overline{\Gamma}^{31}_{im} & 0 & 0 & \varepsilon_{i2}\Phi(\overline{\mathcal{R}}_i) & * & * & \cdots & * & * & * \\ \overline{\Gamma}^{41}_{im} & 0 & 0 & 0 & \varepsilon_{i3}\Phi(\overline{\mathcal{R}}_i) & * & \cdots & * & * & * \\ \overline{\Gamma}^{51}_{im} & 0 & 0 & 0 & 0 & \varepsilon_{i4}\Phi(\overline{\mathcal{R}}_i) & \cdots & * & * & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \overline{\Gamma}^{61}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & \varepsilon_{i,M+3}\Phi(\overline{\mathcal{R}}_i) & * & * \\ \overline{\Gamma}^{71}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -\check{\overline{\mathcal{R}}}_i & * \\ \overline{\Gamma}^{81}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\hat{\overline{\mathcal{R}}}_i \end{bmatrix} < 0, \tag{32}$$

$$-\overline{Q}_i + \textstyle\sum_{l\in\bigwedge_{m,k}} \overline{\pi}_{ml}(h)(\overline{Q}_{il} - \overline{Q}_{ij}) < 0, \tag{33}$$

$$-\overline{Q}_i + \textstyle\sum_{l\in\bigwedge_{m,k}} \underline{\pi}ml(h)(\overline{Q}_{il} - \overline{Q}_{ij}) < 0, \tag{34}$$

where

$$\overline{\Xi}_{im,\overline{m}} = \begin{bmatrix} \overline{\Xi}^{11}_{im,\overline{m}} & * & * & * & * & * & * & \cdots & * \\ 0 & \overline{\Xi}^{22}_{im,\overline{m}} & * & * & * & * & * & \cdots & * \\ \overline{\Xi}^{31}_{im} & 0 & \overline{\Xi}^{33}_{im} & * & * & * & * & \cdots & * \\ \overline{\Xi}^{41}_{im} & 0 & \overline{\Xi}^{43}_{im} & \overline{\Xi}^{44}_{im} & * & * & * & \cdots & * \\ \overline{\Xi}^{51}_{im} & 0 & \overline{\Xi}^{53}_{im} & \overline{\Xi}^{54}_{im} & \overline{\Xi}^{55}_{im} & * & * & \cdots & * \\ \overline{\Xi}^{61}_{im} & 0 & \overline{\Xi}^{63}_{im} & -\overline{S}_{4i} & \overline{S}_{4i} & \overline{\Xi}^{66}_{im} & * & \cdots & * \\ \overline{\Xi}^{71}_{im} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}^{77}_{im} & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{\Xi}^{M+6,1}_{im} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \overline{\Xi}^{M+6,M+6}_{im} \end{bmatrix},$$

$$\overline{\Xi}_{im,\underline{m}}=\begin{bmatrix}
\overline{\Xi}^{11}_{im,\underline{m}} & * & * & * & * & * & * & \cdots & * \\
0 & \overline{\Xi}^{22}_{im,\underline{m}} & * & * & * & * & * & \cdots & * \\
\overline{\Xi}^{31}_{im} & 0 & \overline{\Xi}^{33}_{im} & * & * & * & * & \cdots & * \\
\overline{\Xi}^{41}_{im} & 0 & \overline{\Xi}^{43}_{im} & \overline{\Xi}^{44}_{im} & * & * & * & \cdots & * \\
\overline{\Xi}^{51}_{im} & 0 & \overline{\Xi}^{53}_{im} & \overline{\Xi}^{54}_{im} & \overline{\Xi}^{55}_{im} & * & * & \cdots & * \\
\overline{\Xi}^{61}_{im} & 0 & \overline{\Xi}^{63}_{im} & -\overline{\mathcal{S}}_{4i} & \overline{\mathcal{S}}_{4i} & \overline{\Xi}^{66}_{im} & * & \cdots & * \\
\overline{\Xi}^{71}_{im} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}^{77}_{im} & \cdots & * \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\overline{\Xi}^{M+6,1}_{im} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \overline{\Xi}^{M+6,M+6}_{im}
\end{bmatrix},$$

$$\overline{\Xi}^{11}_{im,\overline{m}}=sym\{\mathcal{A}_{im}X_{im}\}+\overline{Q}_{im}+\tau_M\overline{Q}_i-4(1+\mu_i)\overline{\mathcal{R}}_i+\sum_{l\in\wedge_{m,k}}\overline{\pi}_{ml}(h)(X_{il}-X_{ij}),$$

$$\overline{\Xi}^{22}_{im,\overline{m}}=sym\{\mathcal{A}_{im}X_{im}\}+\sum_{l\in\wedge_{m,k}}\overline{\pi}_{ml}(h)(X_{il}-X_{ij}),$$

$$\overline{\Xi}^{11}_{im,\underline{m}}=sym\{\mathcal{A}_{im}X_{im}\}+\overline{Q}_{im}+\tau_M\overline{Q}_i-4(1+\mu_i)\overline{\mathcal{R}}_i+\sum_{l\in\wedge_{m,k}}\underline{\pi}_{ml}(h)(X_{il}-X_{ij}),$$

$$\overline{\Xi}^{22}_{im,\underline{m}}=sym\{\mathcal{A}_{im}X_{im}\}+\sum_{l\in\wedge_{m,k}}\underline{\pi}_{ml}(h)(X_{il}-X_{ij}).$$

Case 2. If $\wedge_{m,k}\neq\emptyset$ and $\wedge_{m,uk}\neq\emptyset$, $m\in\wedge_{m,uk}$, for$\forall j\in\wedge_{m,uk}$, we have

$$\begin{bmatrix}
\ddot{\Xi}_{im,\overline{m}} & * & * & * & * & * & \cdots & * & * & * \\
\overline{\Gamma}^{11}_{im} & \overline{\Gamma}^{12}_{im} & * & * & * & * & \cdots & * & * & * \\
\overline{\Gamma}^{21}_{im} & 0 & \Phi(\overline{\mathcal{R}}_i) & * & * & * & \cdots & * & * & * \\
\overline{\Gamma}^{31}_{im} & 0 & 0 & \varepsilon_{i2}\Phi(\overline{\mathcal{R}}_i) & * & * & \cdots & * & * & * \\
\overline{\Gamma}^{41}_{im} & 0 & 0 & 0 & \varepsilon_{i3}\Phi(\overline{\mathcal{R}}_i) & * & \cdots & * & * & * \\
\overline{\Gamma}^{51}_{im} & 0 & 0 & 0 & 0 & \varepsilon_{i4}\Phi(\overline{\mathcal{R}}_i) & \cdots & * & * & * \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\overline{\Gamma}^{61}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & \varepsilon_{i,M+3}\Phi(\overline{\mathcal{R}}_i) & * & * \\
\overline{\Gamma}^{71}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -\check{\overline{\mathcal{R}}}_i & * \\
\overline{\Gamma}^{81}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\hat{\overline{\mathcal{R}}}_i
\end{bmatrix}<0, \tag{35}$$

$$\begin{bmatrix}
\ddot{\Xi}_{im,\underline{m}} & * & * & * & * & * & \cdots & * & * & * \\
\overline{\Gamma}^{11}_{im} & \overline{\Gamma}^{12}_{im} & * & * & * & * & \cdots & * & * & * \\
\overline{\Gamma}^{21}_{im} & 0 & \Phi(\overline{\mathcal{R}}_i) & * & * & * & \cdots & * & * & * \\
\overline{\Gamma}^{31}_{im} & 0 & 0 & \varepsilon_{i2}\Phi(\overline{\mathcal{R}}_i) & * & * & \cdots & * & * & * \\
\overline{\Gamma}^{41}_{im} & 0 & 0 & 0 & \varepsilon_{i3}\Phi(\overline{\mathcal{R}}_i) & * & \cdots & * & * & * \\
\overline{\Gamma}^{51}_{im} & 0 & 0 & 0 & 0 & \varepsilon_{i4}\Phi(\overline{\mathcal{R}}_i) & \cdots & * & * & * \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\overline{\Gamma}^{61}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & \varepsilon_{i,M+3}\Phi(\overline{\mathcal{R}}_i) & * & * \\
\overline{\Gamma}^{71}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -\check{\overline{\mathcal{R}}}_i & * \\
\overline{\Gamma}^{81}_{im} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\hat{\overline{\mathcal{R}}}_i
\end{bmatrix}<0, \tag{36}$$

$$X_{im} - X_{ij} \geq 0, \overline{Q}_{im} - \overline{Q}_{ij} \geq 0, \tag{37}$$

$$-\overline{Q}_i + \textstyle\sum_{l \in \wedge_{m,k}} \overline{\pi}_{ml}(h)(\overline{Q}_{il} - \overline{Q}_{ij}) < 0, \tag{38}$$

$$-\overline{Q}_i + \textstyle\sum_{l \in \wedge_{m,k}} \underline{\pi}_{ml}(h)(\overline{Q}_{il} - \overline{Q}_{ij}) < 0, \tag{39}$$

where

$$\ddot{\Xi}_{im,\overline{m}} = \begin{bmatrix} \ddot{\Xi}^{11}_{im,\overline{m}} & * & * & * & * & * & * & \cdots & * \\ 0 & \ddot{\Xi}^{22}_{im,\overline{m}} & * & * & * & * & * & \cdots & * \\ \overline{\Xi}^{31}_{im} & 0 & \overline{\Xi}^{33}_{im} & * & * & * & * & \cdots & * \\ \overline{\Xi}^{41}_{im} & 0 & \overline{\Xi}^{43}_{im} & \overline{\Xi}^{44}_{im} & * & * & * & \cdots & * \\ \overline{\Xi}^{51}_{im} & 0 & \overline{\Xi}^{53}_{im} & \overline{\Xi}^{54}_{im} & \overline{\Xi}^{55}_{im} & * & * & \cdots & * \\ \overline{\Xi}^{61}_{im} & 0 & \overline{\Xi}^{63}_{im} & -\overline{\mathcal{S}}_{4i} & \overline{\mathcal{S}}_{4i} & \overline{\Xi}^{66}_{im} & * & \cdots & * \\ \overline{\Xi}^{71}_{im} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}^{77}_{im} & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{\Xi}^{M+6,1}_{im} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \overline{\Xi}^{M+6,M+6}_{im} \end{bmatrix},$$

$$\ddot{\Xi}_{im,\underline{m}} = \begin{bmatrix} \ddot{\Xi}^{11}_{im,\underline{m}} & * & * & * & * & * & * & \cdots & * \\ 0 & \ddot{\Xi}^{22}_{im,\underline{m}} & * & * & * & * & * & \cdots & * \\ \overline{\Xi}^{31}_{im} & 0 & \overline{\Xi}^{33}_{im} & * & * & * & * & \cdots & * \\ \overline{\Xi}^{41}_{im} & 0 & \overline{\Xi}^{43}_{im} & \overline{\Xi}^{44}_{im} & * & * & * & \cdots & * \\ \overline{\Xi}^{51}_{im} & 0 & \overline{\Xi}^{53}_{im} & \overline{\Xi}^{54}_{im} & \overline{\Xi}^{55}_{im} & * & * & \cdots & * \\ \overline{\Xi}^{61}_{im} & 0 & \overline{\Xi}^{63}_{im} & -\overline{\mathcal{S}}_{4i} & \overline{\mathcal{S}}_{4i} & \overline{\Xi}^{66}_{im} & * & \cdots & * \\ \overline{\Xi}^{71}_{im} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}^{77}_{im} & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{\Xi}^{M+6,1}_{im} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \overline{\Xi}^{M+6,M+6}_{im} \end{bmatrix},$$

$$\ddot{\Xi}^{11}_{im,\overline{m}} = sym\{\mathcal{A}_{im} X_{im}\} + \overline{Q}_{im} + \tau_M \overline{Q}_i - 4(1+\mu_i)\overline{\mathcal{R}}_i + \sum_{l \in \wedge_{m,k}} \overline{\pi}_{ml}(h)(X_{il} - X_{ij}) + \overline{\pi}_{mm}(h)(X_{im} - X_{ij}),$$

$$\ddot{\Xi}^{22}_{im,\overline{m}} = sym\{\mathcal{A}_{im} X_{im}\} + \sum_{l \in \wedge_{m,k}} \overline{\pi}_{ml}(h)(X_{il} - X_{ij}) + \overline{\pi}_{mm}(h)(X_{im} - X_{ij}),$$

$$\ddot{\Xi}^{11}_{im,\underline{m}} = sym\{\mathcal{A}_{im} X_{im}\} + \overline{Q}_{im} + \tau_M \overline{Q}_i - 4(1+\mu_i)\overline{\mathcal{R}}_i + \sum_{l \in \wedge_{m,k}} \underline{\pi}_{ml}(h)(X_{il} - X_{ij}) + \underline{\pi}_{mm}(h)(X_{im} - X_{ij}),$$

$$\ddot{\Xi}^{22}_{im,\underline{m}} = sym\{\mathcal{A}_{im} X_{im}\} + \sum_{l \in \wedge_{m,k}} \underline{\pi}_{ml}(h)(X_{il} - X_{ij}) + \underline{\pi}_{mm}(h)(X_{im} - X_{ij}).$$

Case 3. If $\bigwedge_{m,k} = \emptyset$, $\bigwedge_{m,uk} \neq \emptyset$, $m \in \bigwedge_{m,uk}$, and there exist $l \neq m$ and $l \in \bigwedge_{m,uk}$, we have

$$\begin{bmatrix} \breve{\Xi}_{im,m} & * & * & * & * & * & \cdots & * & * & * \\ \overline{\Gamma}_{im}^{11} & \overline{\Gamma}_{im}^{12} & * & * & * & * & \cdots & * & * & * \\ \overline{\Gamma}_{im}^{21} & 0 & \Phi(\overline{\mathcal{R}}_i) & * & * & * & \cdots & * & * & * \\ \overline{\Gamma}_{im}^{31} & 0 & 0 & \varepsilon_{i2}\Phi(\overline{\mathcal{R}}_i) & * & * & \cdots & * & * & * \\ \overline{\Gamma}_{im}^{41} & 0 & 0 & 0 & \varepsilon_{i3}\Phi(\overline{\mathcal{R}}_i) & * & \cdots & * & * & * \\ \overline{\Gamma}_{im}^{51} & 0 & 0 & 0 & 0 & \varepsilon_{i4}\Phi(\overline{\mathcal{R}}_i) & \cdots & * & * & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \overline{\Gamma}_{im}^{61} & 0 & 0 & 0 & 0 & 0 & \cdots & \varepsilon_{i,M+3}\Phi(\overline{\mathcal{R}}_i) & * & * \\ \overline{\Gamma}_{im}^{71} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -\check{\overline{\mathcal{R}}}_i & * \\ \overline{\Gamma}_{im}^{81} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & -\hat{\overline{\mathcal{R}}}_i \end{bmatrix} < 0, \tag{40}$$

where

$$\breve{\Xi}_{im,m} = \begin{bmatrix} \breve{\Xi}_{im,m}^{11} & * & * & * & * & * & * & \cdots & * \\ 0 & \breve{\Xi}_{im,m}^{22} & * & * & * & * & * & \cdots & * \\ \overline{\Xi}_{im}^{31} & 0 & \overline{\Xi}_{im}^{33} & * & * & * & * & \cdots & * \\ \overline{\Xi}_{im}^{41} & 0 & \overline{\Xi}_{im}^{43} & \overline{\Xi}_{im}^{44} & * & * & * & \cdots & * \\ \overline{\Xi}_{im}^{51} & 0 & \overline{\Xi}_{im}^{53} & \overline{\Xi}_{im}^{54} & \overline{\Xi}_{im}^{55} & * & * & \cdots & * \\ \overline{\Xi}_{im}^{61} & 0 & \overline{\Xi}_{im}^{63} & -\overline{\mathcal{S}}_{4i} & \overline{\mathcal{S}}_{4i} & \overline{\Xi}_{im}^{66} & * & \cdots & * \\ \overline{\Xi}_{im}^{71} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}_{im}^{77} & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{\Xi}_{im}^{M+6,1} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \overline{\Xi}_{im}^{M+6,M+6} \end{bmatrix},$$

$$\breve{\Xi}_{im,m}^{11} = sym\{\mathcal{A}_{im}X_{im}\} + \overline{Q}_{im} + \tau_M\overline{Q}_i - 4(1+\mu_i)\overline{\mathcal{R}}_i + a_m\pi_{ll}(h)(X_{im} - X_{ij}),$$
$$\breve{\Xi}_{im,m}^{22} = sym\{\mathcal{A}_{im}X_{im}\} + a_m\pi_{ll}(h)(X_{im} - X_{ij}).$$

In addition, the other scalars are given as follows

$$
\begin{aligned}
&\overline{\Xi}_{im}^{31} = \sum_{q=1}^{M}(Y_{im}^{q})^{\mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}} - 2(1+\mu_i)\overline{\mathcal{R}}_i - \overline{\mathcal{S}}_{1i} - \overline{\mathcal{S}}_{2i} - \overline{\mathcal{S}}_{3i} - \overline{\mathcal{S}}_{4i},\\
&\overline{\Xi}_{im}^{33} = -2(4+\mu_i)\overline{\mathcal{R}}_i + 2\overline{\mathcal{S}}_{1i} - 2\overline{\mathcal{S}}_{2i} + 2\overline{\mathcal{S}}_{3i} - 2\overline{\mathcal{S}}_{4i}, \overline{\Xi}_{im}^{41} = \overline{\mathcal{S}}_{1i} + \overline{\mathcal{S}}_{2i} - \overline{\mathcal{S}}_{3i} - \overline{\mathcal{S}}_{4i},\\
&\overline{\Xi}_{im}^{43} = -2(2-\mu_i)\overline{\mathcal{R}}_i - \overline{\mathcal{S}}_{1i} + \overline{\mathcal{S}}_{2i} + \overline{\mathcal{S}}_{3i} - \overline{\mathcal{S}}_{4i}, \overline{\Xi}_{im}^{44} = -\overline{Q}_{im} - 4(2-\mu_i)\overline{\mathcal{R}}_i,\\
&\overline{\Xi}_{im}^{51} = \overline{\mathcal{S}}_{3i} + \overline{\mathcal{S}}_{4i}, \tilde{\Xi}_{im}^{53} = -\overline{\mathcal{S}}_{3i} + \overline{\mathcal{S}}_{4i} + 3(2-\mu_i)\overline{\mathcal{R}}_i, \overline{\Xi}_{im}^{54} = 3(2-\mu_i)\overline{\mathcal{R}}_i, \overline{\Xi}_{im}^{55} = -3(2-\mu_i)\overline{\mathcal{R}}_i,\\
&\overline{\Xi}_{im}^{61} = 3(1+\mu_i)\overline{\mathcal{R}}_i, \overline{\Xi}_{im}^{62} = \overline{\mathcal{S}}_{2i} + 3(1+\mu_i)\overline{\mathcal{R}}_i, \overline{\Xi}_{im}^{66} = -3(1+\mu_i)\overline{\mathcal{R}}_i,\\
&\overline{\Xi}_{im}^{71} = Y_{im}^{1\ \mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}, \overline{\Xi}_{im}^{77} = -\theta_0\gamma_1\overline{\Omega}_i, \overline{\Xi}_{im}^{M+6,1} = Y_{im}^{M\ \mathrm{T}}\mathcal{B}_{im}^{\mathrm{T}}, \overline{\Xi}_{im}^{M+6,M+6} = -\theta_0\gamma_M\overline{\Omega}_i,\\
&\overline{\Gamma}_{im}^{11} = \begin{bmatrix} X_{im}\mathcal{G}_{ijm}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix},\\
&\overline{\Gamma}_{im}^{12} = -(N-1)^{-1}diag\{\varepsilon_{i1}^{-1}X_{im}, \cdots, \varepsilon_{j1}^{-1}X_{im, j\neq i}, \cdots, \varepsilon_{N1}^{-1}X_{im}\},\\
&\overline{\Gamma}^{21} = \tau_M\begin{bmatrix} \mathcal{A}_{im}X_{im} & 0 & \mathcal{B}_{im}\sum_{q=1}^{M}Y_{im}^{q} & 0 & 0 & 0 & \mathcal{B}_{im}Y_{im}^{1} & \cdots & \mathcal{B}_{im}Y_{im}^{M} \end{bmatrix},\\
&\overline{\Gamma}_{im}^{31} = \tau_M\begin{bmatrix} \mathcal{A}_{im}X_{im} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix},\\
&\overline{\Gamma}_{im}^{41} = \tau_M\begin{bmatrix} 0 & 0 & \mathcal{B}_{im}\sum_{q=1}^{M}Y_{im}^{q} & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix},\\
&\overline{\Gamma}_{im}^{51} = \tau_M\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \mathcal{B}_{im}Y_{im}^{1} & \cdots & 0 \end{bmatrix},\\
&\overline{\Gamma}_{im}^{61} = \tau_M\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \mathcal{B}_{im}Y_{im}^{M} \end{bmatrix},\\
&\overline{\Gamma}_{im}^{71} = \tau_M\begin{bmatrix} \mathcal{G}_{ijm}X_{im} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix},
\end{aligned}
$$

$$
\begin{aligned}
&\check{\overline{\mathcal{R}}}_i = -(N-1)^{-1}diag\{(1+\varepsilon_{i2}+\varepsilon_{i3}+\cdots+\varepsilon_{i,M+3})^{-1}\Phi(\overline{\mathcal{R}}_1), \cdots, (1+\varepsilon_{j2}+\varepsilon_{j3}+\cdots+\varepsilon_{j,M+3})^{-1}\Phi(\overline{\mathcal{R}}_j), j\neq i,\\
&\cdots, (1+\varepsilon_{N2}+\varepsilon_{N3}+\cdots+\varepsilon_{N,M+3})^{-1}\Phi(\overline{\mathcal{R}}_N)\},\\
&\Phi(\overline{\mathcal{R}}_l) = -2\alpha_i X_{im} + \alpha_i^2\overline{R}_l, l = \{1, \cdots, j(j\neq i), \cdots, N\},\\
&\overline{\Gamma}_{im}^{81} = \begin{bmatrix} 0 & 0 & \overline{\mathcal{S}}_{1i}+\overline{\mathcal{S}}_{3i} & -\overline{\mathcal{S}}_{1i}+\overline{\mathcal{S}}_{3i} & -\overline{\mathcal{S}}_{3i} & 0 & 0 & \cdots & 0\\ 0 & 0 & \overline{\mathcal{S}}_{2i}+\overline{\mathcal{S}}_{4i} & -\overline{\mathcal{S}}_{2i}+\overline{\mathcal{S}}_{4i} & -\overline{\mathcal{S}}_{4i} & 0 & 0 & \cdots & 0\\ \overline{\mathcal{S}}_{1i}+\overline{\mathcal{S}}_{3i} & 0 & -\overline{\mathcal{S}}_{1i}+\overline{\mathcal{S}}_{3i} & 0 & 0 & -\overline{\mathcal{S}}_{3i} & 0 & \cdots & 0\\ \overline{\mathcal{S}}_{2i}+\overline{\mathcal{S}}_{4i} & 0 & -\overline{\mathcal{S}}_{2i}+\overline{\mathcal{S}}_{4i} & 0 & 0 & -\overline{\mathcal{S}}_{4i} & 0 & \cdots & 0 \end{bmatrix},\\
&\hat{\overline{\mathcal{R}}}_i = diag\{-(1-\mu_i)^{-1}\overline{\mathcal{R}}_i, -3(1-\mu_i)^{-1}\overline{\mathcal{R}}_i, -\mu_i^{-1}\overline{\mathcal{R}}_i, -3\mu_i^{-1}\overline{\mathcal{R}}_i\}.
\end{aligned}
$$

Furthermore, the memory controller gain matrix is $K_{im}^{q} = Y_{im}^{q}X_{im}^{-1}$.

Proof: Define $X_{im} = \mathcal{P}_{im}^{-1}$, then pre-multiplying and post-multiplying (15),(17),(20) with $diag\{X_{im}, \cdots, X_{im}, \mathcal{R}_i^{-1}$ $\mathcal{R}_i^{-1}, \mathcal{R}_i^{-1}, \mathcal{R}_i^{-1}, \mathcal{R}_i^{-1}, \mathcal{R}_i^{-1}, X_{im}, X_{im}, X_{im}, X_{im}\}$ respectively, and define $Y_{im} = K_{im}X_{im}, \overline{\mathcal{R}}_{im} = X_{im}\mathcal{R}_{im}X_{im}, \overline{Q}_{im} = X_{im}Q_{im}X_{im}, \overline{\mathcal{S}}_{1i} = X_{im}\mathcal{S}_{1i}X_{im}, \overline{\mathcal{S}}_{2i} = X_{im}\mathcal{S}_{2i}X_{im}, \overline{\mathcal{S}}_{3i} = X_{im}\mathcal{S}_{3i}X_{im}, \overline{\mathcal{S}}_{4i} = X_{im}\mathcal{S}_{4i}X_{im}, \overline{Q}_i = X_{im}Q_iX_{im}, \overline{\Omega}_i = X_{im}\Omega_iX_{im}$. According to Schur complement and Lemma 2, and considering $\pi_{ml}(h) \in [\underline{\pi}_{ml}(h), \overline{\pi}_{ml}(h)]$, (31), (32), (35), (36), (40) can be obtained. This completes the proof.

Remark 4. For a given dwell time h, the TR $\pi_{ml}(h)$ can be regarded as a linear combination of its upper and lower bounds, namely $\pi_{ml}(h) = \beta_1\overline{\pi}_{ml}(h) + \beta_2\underline{\pi}_{ml}(h)$, where $\beta_1 + \beta_2 = 1$ and $\beta_1 > 0, \beta_2 > 0$. We can change β_1 and β_2 to get the corresponding value of $\pi_{ml}(h)$.

Remark 5. Theorem 2 presents a solution algorithm in terms of linear matrix inequality to obtain the controller gains. However, the transition probability in Case 3 is completely unknown. We introduce $a_m\pi_{ll}(h)$ to replace the unknown $\pi_{mm}(h)$, in which a_m is a parameter to be estimated. If this case exists, we can obtain the estimation value of parameter a_m by employing the optimization algorithm in [24].

4. SIMULATION EXAMPLE

This section will give an example to verify the feasibility of the above theoretical results. We consider a semi-Markovian interconnected system composed of three subsystems, and the relevant parameters are as follows [23]:

$$\mathcal{A}_{11} = \begin{bmatrix} 0 & 5 \\ -9.81 & -1 \end{bmatrix}, \mathcal{A}_{12} = \begin{bmatrix} 0 & 5 \\ -0.31 & -1 \end{bmatrix}, \mathcal{A}_{21} = \begin{bmatrix} 0 & 5 \\ -9.81 & -1.4 \end{bmatrix}, \mathcal{A}_{22} = \begin{bmatrix} 0 & 5 \\ -0.31 & -1.4 \end{bmatrix},$$
$$\mathcal{A}_{31} = \begin{bmatrix} 0 & 5 \\ -9.81 & -0.5 \end{bmatrix}, \mathcal{A}_{32} = \begin{bmatrix} 0 & 5 \\ -0.31 & -0.5 \end{bmatrix}, \mathcal{B}_{11} = \mathcal{B}_{12} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \mathcal{B}_{21} = \mathcal{B}_{22} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix},$$
$$\mathcal{B}_{31} = \mathcal{B}_{32} = \begin{bmatrix} 0 \\ 0.33 \end{bmatrix}, \mathcal{G}_{121} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \mathcal{G}_{122} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \mathcal{G}_{211} = \begin{bmatrix} 0 & 0 \\ 0.8 & 0 \end{bmatrix}, \mathcal{G}_{212} = \begin{bmatrix} 0 & 0 \\ 0.8 & 0 \end{bmatrix},$$
$$\mathcal{G}_{311} = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}, \mathcal{G}_{312} = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}.$$

The transition probability matrix of system (10) is $\begin{bmatrix} ? & ? \\ \pi_{21}(h) & \pi_{22}(h) \end{bmatrix}$, where "?" represents a completely unknown transition probability, and $\pi_{21}(h) \in [0.4, 0.7], \pi_{22}(h) \in [-0.6, -0.3]$. The scalars and positive matrix are chosen as $\tau_M = 0.1, a_m = 0.5, \gamma = 0.8, \varepsilon_{i1} = \varepsilon_{i2} = \varepsilon_{i3} = \varepsilon_{i4} = 1, \gamma_q = 20$ and $R_i = I$, respectively. By solving LMIs in Theorem 2, the weighting matrices in equation (4) are $\Omega_1 = 0.3844, \Omega_2 = 0.3839, \Omega_3 = 0.3792$, and the corresponding controller gains are

$$\begin{aligned}
K_{11}^1 &= \begin{bmatrix} 0.1456 & -0.2717 \end{bmatrix}, K_{11}^2 = \begin{bmatrix} -0.2089 & -0.3153 \end{bmatrix}, \\
K_{12}^1 &= \begin{bmatrix} -3.5900 & -15.3939 \end{bmatrix}, K_{12}^2 = \begin{bmatrix} -3.6974 & -15.8393 \end{bmatrix}, \\
K_{21}^1 &= \begin{bmatrix} 0.1452 & -0.2697 \end{bmatrix}, K_{21}^2 = \begin{bmatrix} -0.2548 & -0.3342 \end{bmatrix}, \\
K_{22}^1 &= \begin{bmatrix} -2.6929 & -12.1132 \end{bmatrix}, K_{22}^2 = \begin{bmatrix} -2.7779 & -12.4794 \end{bmatrix}, \\
K_{31}^1 &= \begin{bmatrix} 0.2304 & -0.2927 \end{bmatrix}, K_{31}^2 = \begin{bmatrix} -0.1659 & -0.3184 \end{bmatrix}, \\
K_{32}^1 &= \begin{bmatrix} -1.9339 & -12.7768 \end{bmatrix}, K_{32}^2 = \begin{bmatrix} -1.9851 & -13.0934 \end{bmatrix}.
\end{aligned}$$

The initial conditions are given as $x_1(0) = \begin{bmatrix} -0.45 & 0.85 \end{bmatrix}^{\mathrm{T}}, x_2(0) = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix}^{\mathrm{T}}, x_3(0) = \begin{bmatrix} -0.95 & 0.55 \end{bmatrix}^{\mathrm{T}}$. Figure 2 shows the states response of dynamic METM with M=2. To illustrate the effectiveness of the designed method, Figure 3 presents the state's response without control input. Figure 4 plots the control input of the systems. Figure 5 shows the switching states of the semi-Markovian process. Figure 6 depicts the data-releasing instants and intervals of dynamic METM with M=2. Figure 7 describes the instants and intervals of memoryless ETM (the case of M = 1). From Figure 6 and Figure 7, we can see that the event-triggered times for M=2 are significantly less than the case of M=1, which illustrates that the dynamic METM has more advantages in reducing the number of released signals.

5. CONCLUSION

In this paper, a dynamic METM has been drawn for the decentralized control of interconnected semi-Markovian systems with partially accessible TRs. Considering both the dynamic METM and partially accessible TRs, a new kind of interconnected semi-Markovian system model has been designed. By applying the Lyapunov function theory and the LMI techniques, some sufficient conditions have been obtained to ensure the proposed system is asymptotical stability. Meanwhile, the controller gain matrices and the parameters of dynamic METM are also solved simultaneously. Finally, a simulation example has been used to verify the effectiveness

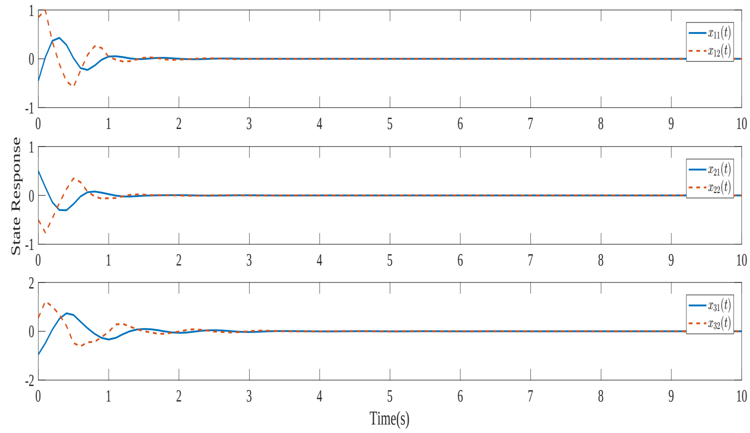

Figure 2. The state's response of dynamic METM with M = 2. METM: memory event-triggered mechanism.

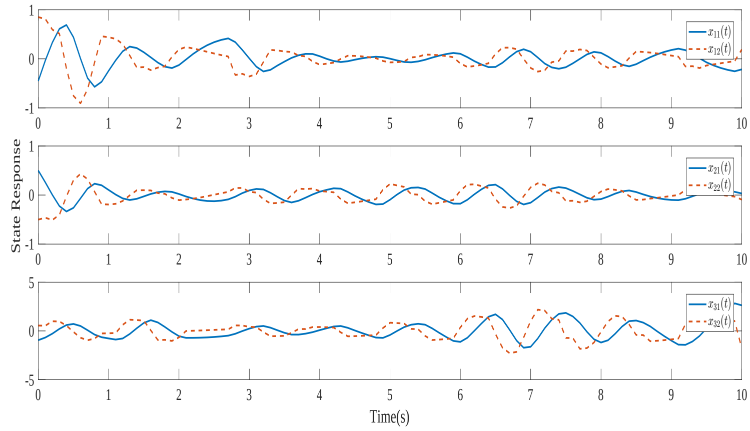

Figure 3. The state's response without control input.

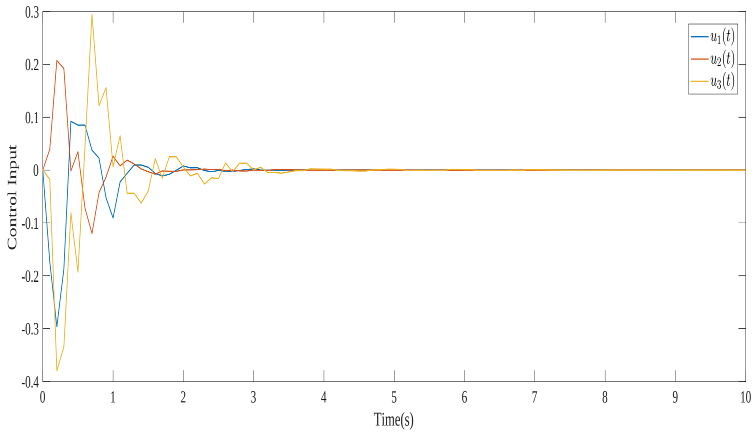

Figure 4. The control input of the three systems.

of the developed method, which illustrates the proposed dynamic METM can save more network resources

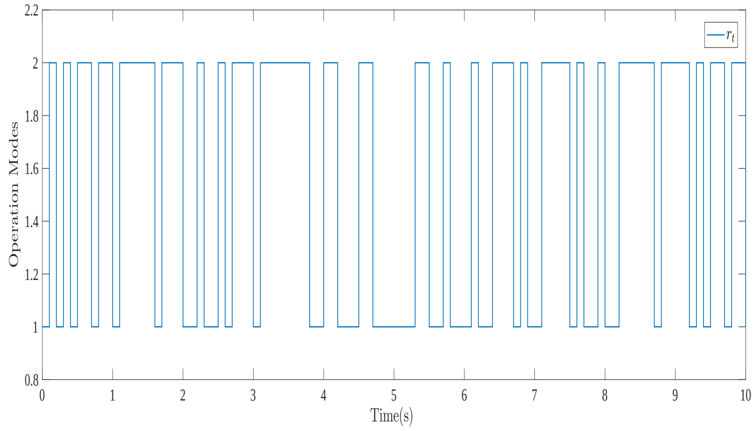

Figure 5. The switching state of the semi-Markovian process.

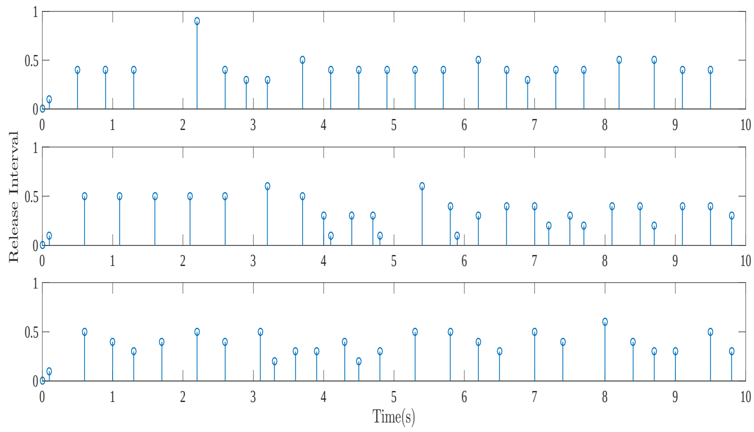

Figure 6. The data-releasing instants and intervals of dynamic METM with M = 2. METM: memory event-triggered mechanism.

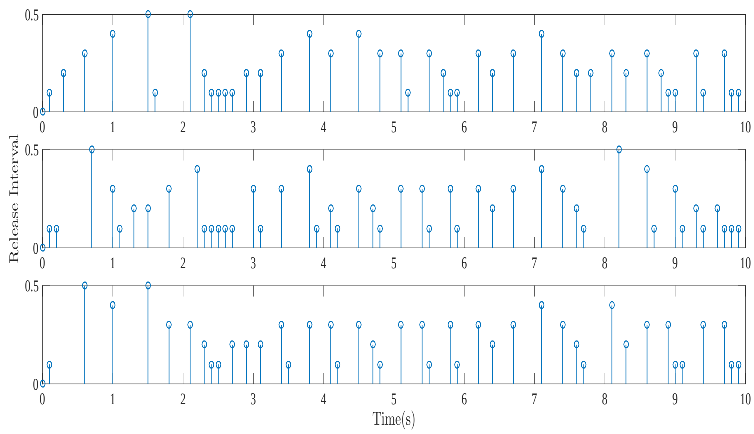

Figure 7. The instants and intervals of memoryless ETM (the case of M = 1). ETM: event-triggered mechanism.

than the memoryless event-triggered mechanism.

In the future, we will pay attention to the distributed memory event-triggered control design for interconnected systems via the observer method. Moreover, the memory event-triggered security control problem for the interconnected Markovian jump systems with cyber attacks is also an interesting issue, which is left to be developed.

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Authors' contributions

Made substantial contributions to the conception and design of the study and performed data analysis and interpretation: Tan Y, Cheng X, Li X, Bai J

Performed data acquisition, as well as providing administrative, technical, and material support: Tan Y, Liu J

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Consent for publication

Not applicable.

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