Research Article

Backstepping-based state estimation for a class of stochastic nonlinear systems

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Abstract

The state estimation problem is investigated for a class of continuous-time stochastic nonlinear systems, where a novel filter design method is proposed based on backstepping design and stochastic differential equation. In particular, the structure of the filter is developed following the nonlinear system model, and then the estimation error dynamics can be described by a stochastic differential equation. Motivated by backstepping procedure, the nonlinear dynamics can be converted to an Ornstein–Uhlenbeck process via the control loop design. Thus, the estimation can be achieved once the estimation error is bounded and the variance of the error can be optimized. Since the ideal estimation error is a Brownian motion, the filter parameters can be selected following the Lyapunov stability theory and variance assignment method. Following the same framework, the multivariate stochastic systems can be handled with the block backstepping design. To validate the presented design approach, a numerical example is given as the simulation results to demonstrate the state estimation performance.

Keywords: Continuous-time stochastic systems, stochastic differential equation, Ornstein–Uhlenbeck process, backstepping

1. INTRODUCTION

Since state space has been widely used to present the model of the dynamic system, state estimation is a key research problem to characterize the system properties as the internal states are mostly unmeasurable. It is a challenging technical problem because the unmeasurable states are normally subjected to random noises. For example, process noise and measurement noise widely exist in the state space model. Therefore, it is also a filtering problem for state estimation if the random noises have been considered in the design procedure. Nowadays, state estimation and filtering have been adopted in many applications such as the robotics system[1], intelligent manufacturing[2], transportation monitoring[3], industrial performance optimization[4,5], *etc*.

As a well-developed solution for estimation problem, Kalman[6] filter was firstly designed in the 1960s, and it has been considered as a standard design method. However, the traditional Kalman filter was given for a discrete-time linear dynamic system. To deal with the nonlinearities of the system dynamics, many variants have been proposed successfully based on Kalman filter framework, such as extended Kalman filter[7] and unscented Kalam filter[8]. However, the problem formulation for these filters still use discrete-time models. To deal with the continuous-time model, the Kalman-Bucy filter[9] was proposed by solving linear Riccati equation. Notice that the Kalman-Bucy filter has a linear structure. It is difficult to extend the result to nonlinear systems directly, while it is difficult to solve the associated Riccati equation analytically for the nonlinear system model. Following the Kalman filter framework, robust Kalman filter[10] and unscented Kalman filtering[11] have been extended to continuous-time models. Different from the Kalman filter framework, the numerical solution has been developed as particle filter[12], which considers the distribution of the particle for each sampling instant. The main issue of particle filter is the convergence analysis. It is still an open question currently. The minimum entropy filtering has also been presented[13,14], which considers non-Gaussian noises in the system model. Inspired by the probability density function (PDF) in particle filtering and the minimum entropy filtering, the Fokker-Planck-Kolmogorov (FPK) equation is used in this paper to produce the continuous-time solution.

FPK-based state estimation[15] has been presented recently. However, this paper focuses on the filtering design based on linearization. Trying to eliminate the nonlinear effects in the closed-loop, in this paper, the backstepping design is adopted for the estimation error dynamics as the system model is represented by stochastic differential equation. In particular, the structure of the filter can be confirmed using the system model. Thus, the dynamics of the estimation error can be produced following the stochastic differential equation. To stabilize the estimation error, the backstepping design is adopted. As a result, the stochastic differential equation in terms of the estimation error can be further converted to a linear Ornstein-Uhlenbeck process[16], while the virtual tracking error in backstepping is close to zero. In the ideal case, the estimation error is subjected to the Brownian motion. Note that the system states are Gaussian due to the fact that the system dynamics are converted to being linear, while the stochastic differential equation is subjected to Brownian motion. Then, we can use variance to characterize the randomness of the estimation error. Following the variance assignment method[17], the desired value of variance results in the optimal parametric selection for the presented filter. In addition, the presented framework can be used for stochastic distribution control[18] where the tracking error can be optimized similarly to the estimation error in this paper. As one possible application, neural membrane potential estimation can be taken into account, as the nonlinear dynamics[19] are affected by complex random neural interaction[20] and the membrane potential is difficult to measure directly. Then, the proposed estimator is considered as a possible solution.

The remainder of the paper is organized as follows. The formulation is given in Section 2 including the system model and some preliminaries. The filter structure design is given in Section 3 with backstepping procedure. The optimal parameter is obtained in Section 4 using the variance assignment. The simulation results are shown in Section 5 to indicate the performance of the presented filtering algorithm. A discussion for multivariate system is given in Section 6, and the conclusions are summarized in Section 7.

2. FORMULATION

To illustrate the main idea of the estimation method, let us start from a simple univariate stochastic nonlinear system which is described by the Itô process:

$$dx_t = f(x_t)\,dt + \sigma\,dW_t,$$
$$y_t = cx_t, \tag{1}$$

where $f(\cdot)$ stands for a known smooth nonlinear function, W_t denotes the Wiener process, and $\sigma > 0$ is given as a real constant. x_t and y_t stand for the system state and system output, respectively. c denotes a real constant. Note that the measurement equation is presented in linear form, which is widely used in practice to describe the property of the sensor.

To deliver the main result, the preliminaries, such as definitions [21] and lemma [22], are recalled here.

Definition 1 *For any given $V(x_t, t) \in \mathscr{C}^{1,2}(\mathbb{R}^n \times \mathbb{R}_+; \mathbb{R}_+)$ associated with the stochastic differential Equation (1), the differential operator $\mathscr{L}$ is defined as follows:*

$$\mathscr{L}V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_t} f(x_t) + \frac{\partial V}{\partial x_t} v_t + \frac{1}{2}Tr\left\{\frac{\partial^2 V}{\partial x_t^2}\sigma^2\right\}. \tag{2}$$

Definition 2 *The solution process $\{x(t), t \geq 0\}$ of the stochastic system in Equation (1) is said to be bounded in probability if $\lim_{c\to\infty}\sup_{0\leq t<\infty} P\{\|x(t)\| > c\} = 0$, where $P\{\cdot\}$ denotes probability operator and c is a real positive constant.*

Lemma 1 *Consider the stochastic nonlinear system model in Equation (1) and assume that $f(x_t)$ is $\mathscr{C}^1$ in the arguments and $f(0)$ is bounded uniformly in t. If there exist nonnegative functions $V(x_t, t) \in \mathscr{C}^{1,2}(\mathbb{R}^n \times \mathbb{R}_+; \mathbb{R}_+)$, $\mu_1(\cdot)$, $\mu_2(\cdot) \in \mathscr{K}_\infty$, constants $c_1 > 0$ and $c_2 \geq 0$, such that*

$$\mu_1(|x_t|) \leq V(x_t, t) \leq \mu_2(|x_t|),$$
$$\mathscr{L}V \leq -c_1 V(x_t, t) + c_2. \tag{3}$$

where $c_2 = 0$ implies that the system is stable probability sense and $c_2 > 0$ means the system is second-moment stable.

3. FILTER STRUCTURE

Based on the system model in Equation (1), the state estimation scheme can be produced by the following filtering structure.

$$d\hat{x}_t = (f(\hat{x}_t) - v_t)\,dt, \tag{4}$$

where $\hat{x}_t$ denotes the estimated system state x_t. v_t stands for the compensative signal for estimating correction.

Since the estimation error is defined as $e_t = x_t - \hat{x}_t$, the dynamics of e_t can be reflected by the following equations.

$$de_t = (\bar{f}_t + v_t)\,dt + \sigma\,dW_t, \tag{5}$$

where $\bar{f}_t = f(x_t) - f(\hat{x}_t)$.

It is shown above that the estimation problem can be further converted as the estimation error stabilization using the signal v_t. Next, we can introduce an integrator into the system motivated by the backstepping design.

$$de_t = (\bar{f}_t + v_t)\,dt + \sigma\,dW_t,$$
$$dv_t = u_t\,dt, \tag{6}$$

where u_t denotes the new filtering compensative signal. Note that the signal v_t in Equation (5) is the integral of the signal u_t.

Following the backstepping design approach, the virtual signal is given first:

$$\phi(e_t) = -\bar{f}_t - \theta e_t, \tag{7}$$

where θ denotes a real positive constant as a design parameter.

To stabilize Equation (6), the virtual error variable is further defined as follows:

$$\begin{aligned} z_t &= v_t - \phi(e_t) \\ &= v_t + \bar{f}_t + \theta e_t. \end{aligned} \tag{8}$$

Substituting the virtual error z_t into Equation (6), we have

$$de_t = (-\theta e_t + z_t)\, dt + \sigma\, dW_t. \tag{9}$$

The Itô's lemma can be used here to obtain the following equation:

$$\begin{aligned} dz_t &= dv_t - d\phi(e_t) \\ &= \left(u_t - (-\theta e_t + z_t)\frac{\partial \phi(e_t)}{\partial e} - \frac{\sigma^2}{2}\frac{\partial^2 \phi(e_t)}{\partial e^2} \right) dt \\ &\quad - \sigma\frac{\partial \phi(e_t)}{\partial e}\, dW_t. \end{aligned} \tag{10}$$

Then, a Lyapunov function candidate is selected as follows:

$$V_t = V_e + V_z = \frac{1}{2}e_t^2 + \frac{1}{4}z_t^4, \tag{11}$$

which leads to

$$\mathscr{L}V_t = \mathscr{L}V_x + \mathscr{L}V_z. \tag{12}$$

Based on Definition 1, Lemma 1, and Young's inequality, the following result is obtained:

$$\begin{aligned} \mathscr{L}V_e &= e_t(-\theta e_t + z_t) + \frac{\sigma^2}{2} \\ &= -\theta e_t^2 + e_t z_t + \frac{\sigma^2}{2} \\ &\leq -\theta e_t^2 + \frac{1}{2}e_t^2 + \frac{1}{2}z_t^2 + \frac{\sigma^2}{2} \\ &= \left(-\theta + \frac{1}{2} \right)e_t^2 + z_t^2 + \frac{\sigma^2}{2} \\ &\leq \left(-\theta + \frac{1}{2} \right)e_t^2 + \frac{1}{2}z_t^4 + \frac{\sigma^2 + 1}{2}, \end{aligned} \tag{13}$$

and

$$\mathscr{L}V_z = z_t^3 \left(u_t - (-\theta e_t + z_t) \frac{\partial \phi(e_t)}{\partial e} - \frac{\sigma^2}{2} \frac{\partial^2 \phi(e_t)}{\partial e^2} \right)$$

$$+ \frac{3\sigma^2}{2} \left(\frac{\partial \phi(e_t)}{\partial e} \right)^2 z_t^2$$

$$\leq z_t^3 \left(u_t + \theta e_t \frac{\partial \phi(e_t)}{\partial e} - z_t \frac{\partial \phi(e_t)}{\partial e} - \frac{\sigma^2}{2} \frac{\partial^2 \phi(e_t)}{\partial e^2} \right)$$

$$+ \frac{1}{2} + \frac{9\sigma^4}{8} \left(\frac{\partial \phi(e_t)}{\partial e} \right)^4 z_t^4$$

$$= z_t^3 \left(u_t + \theta e_t \frac{\partial \phi(e_t)}{\partial e} - \frac{\sigma^2}{2} \frac{\partial^2 \phi(e_t)}{\partial e^2} \right)$$

$$+ \left(\frac{9\sigma^4}{8} \left(\frac{\partial \phi(e_t)}{\partial e} \right)^4 - \frac{\partial \phi(e_t)}{\partial e} \right) z_t^4 + \frac{1}{2}. \tag{14}$$

Thus, the compensative signal u_t is further designed as

$$u_t = -\theta e_t \frac{\partial \phi(e_t)}{\partial e} + \frac{\sigma^2}{2} \frac{\partial^2 \phi(e_t)}{\partial e^2} - G z_t, \tag{15}$$

where G denotes a design parametric function.

Substituting the designed signal u_t to $\mathscr{L}V_z$, Equation (12) can be rewritten as follows:

$$\mathscr{L}V_t \leq \left(-\theta + \frac{1}{2} \right) e_t^2 + \frac{1}{2} z_t^4 + \frac{\sigma^2 + 2}{2} - G z_t^4$$

$$+ \left(\frac{9\sigma^4}{8} \left(\frac{\partial \phi(e_t)}{\partial e} \right)^4 - \frac{\partial \phi(e_t)}{\partial e} \right) z_t^4$$

$$= \left(-\theta + \frac{1}{2} \right) e_t^2 + \frac{\sigma^2 + 2}{2}$$

$$+ \left(\frac{1}{2} - G + \frac{9\sigma^4}{8} \left(\frac{\partial \phi(e_t)}{\partial e} \right)^4 - \frac{\partial \phi(e_t)}{\partial e} \right) z_t^4. \tag{16}$$

Using Lemma 1, the estimation error e_t with the designed signal u_t is bounded in probability sense. To simplify the expression of $\mathscr{L}V_t$, G can be selected as follows, and then the z^4 term can be further eliminated.

$$G = \frac{1}{2} - \frac{9\sigma^4}{8} \left(\frac{\partial \phi(x_t)}{\partial x} \right)^4 + \frac{\partial \phi(x_t)}{\partial x}, \tag{17}$$

which results in

$$\mathscr{L}V_t \leq \left(-\theta + \frac{1}{2} \right) e_t^2 + \frac{\sigma^2 + 2}{2}. \tag{18}$$

Thus, it is shown that the estimation error is bounded in probability sense when $\theta \geq \frac{1}{2}$.

Note that, as the linear dynamical measurement equation is known, the estimation error can be approximated by the measurement error signal.

$$e_t = c^{-1} (y_t - c\hat{x}_t), \tag{19}$$

which indicates that the presented compensative signal is implementable.

4. PARAMETER SELECTION

Substituting the compensative signal into $\mathscr{L}V_z$ shows that the virtual error z_t is also bounded in probability sense. Therefore, the estimation error with the designed signal can be represented by the linear Ornstein-Uhlenbeck process.

$$de_t = -\theta e_t dt + \sigma dW_t. \tag{20}$$

For the obtained Ornstein-Uhlenbeck process, the Fokker-Planck-Kolmogorov equation can be obtained as follows:

$$\frac{\partial p(x,t)}{\partial t} = \theta \frac{\partial}{\partial x}[xp(x,t)] + \frac{\sigma^2}{2}\frac{\partial^2 p(x,t)}{\partial x^2}, \tag{21}$$

where $p(x,t)$ and x stand for the probability density function and the random variable of e_t, respectively.

Solving the presented FPK equation in analytical form, we have

$$p(x,t) = \sqrt{\frac{\theta}{\pi\sigma^2\left(1 - e^{-2t\theta}\right)}} \exp\left(-\frac{\theta}{\sigma^2}\frac{\left(x - x_0 e^{-t\theta}\right)^2}{1 - e^{-2t\theta)}}\right), \tag{22}$$

where x_0 denotes the initial value of e_t at t_0.

Following the aforementioned discussion in introduction, the designed signal governs the PDF of estimation error e_t following the Gaussian distribution. Moreover, the mean value converges to zero and the variance value can be calculated as follows:

$$Var(e_t) = \frac{\sigma^2}{2\theta}\left(1 - e^{-2t\theta}\right), \tag{23}$$

where the variable is governed by the design parameter θ.

To achieve the filtering performance, the design parameter θ should be selected properly. Note that, if we have the ideal case for the system state estimation, $\bar{f}_t = 0$ holds. Thus, the estimation error dynamics can be further described as follows:

$$de_t = \sigma dW_t, \tag{24}$$

where the associated FPK equation is obtained as the following heat equation.

$$\frac{\partial p(x,t)}{\partial t} = \frac{\sigma^2}{2}\frac{\partial^2 p(x,t)}{\partial x^2}. \tag{25}$$

Based on Green's function and heat kernel, the optimal variance $r(t)$ can be obtained.

$$r(t) = \sigma^2 t. \tag{26}$$

Then, the parameter θ can be formulated by analytically solving the following equation.

$$\frac{\sigma^2}{2\theta}\left(1 - e^{-2t\theta}\right) = r(t), \tag{27}$$

which can be rewritten as follows.

$$e^{-2t\theta} = -\frac{2r(t)}{\sigma^2}\theta + 1. \tag{28}$$

Based on Lambert W function, we can further solve this equation and its solution is given as follows:

$$\theta = \frac{W_0\left(-\frac{t\sigma^2}{r(t)}e^{-\frac{t\sigma^2}{r(t)}}\right)}{2t} + \frac{1}{2t}, \tag{29}$$

where $W_0(\cdot)$ stands for Lambert W function.

Note that the real solution of equation above does not always exist, and we can consider the stationary solution to simplify the calculation.

$$\theta = \frac{1}{2t}, t \le 1. \tag{30}$$

We can further select $\theta = \frac{1}{2}$ when $t > 1$ in order to satisfy the estimation error stabilization condition.

Remark 1 *Note that variance is equivalent to entropy for linear stochastic system, thus the proposed filtering algorithm can be considered as a special case for minimum entropy filtering.*

To summarize the design procedure, the following pseudo-code is demonstrated here as Algorithm 1.

Algorithm 1 Backstepping based filtering for stochastic nonlinear systems

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Require: Model of the investigated system f, system discretization for simulation.
Input: Setup simulation time t_s and the measured value y_t
Output: The estimated value of the system states $\hat{x}_t$

    Initialization: Pre-specified the initial values.
    for *Runningtime* $\le t_s$ **do**
        Obtain the filter structure in Equation (4) and obtain the estimation error value
        Convert the system model into backstepping design form using Equation (6)
        Define virtual input [Equation (7)] and formulate the virtual error [Equation (8)]
        if Lambert W function in Equation (29) is solvable **then**
            Select the parameter θ by Equation (29)
        else
            $\theta = 1/2$
        end if
        Obtain the compensative filtering signal by Equation (15)
        Calculate v_t by integral of u_t
        Produce the estimated state values by Equation (4).
    end for

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5. SIMULATION

As a validation, we consider the following numerical system model as an example:

$$dx_t = (-x_t + \sin(x_t))\, dt + 0.01 dW_t,$$
$$y_t = x_t. \tag{31}$$

To achieve the objective of filtering design, the system state x_t subjected to noise should be approximated using
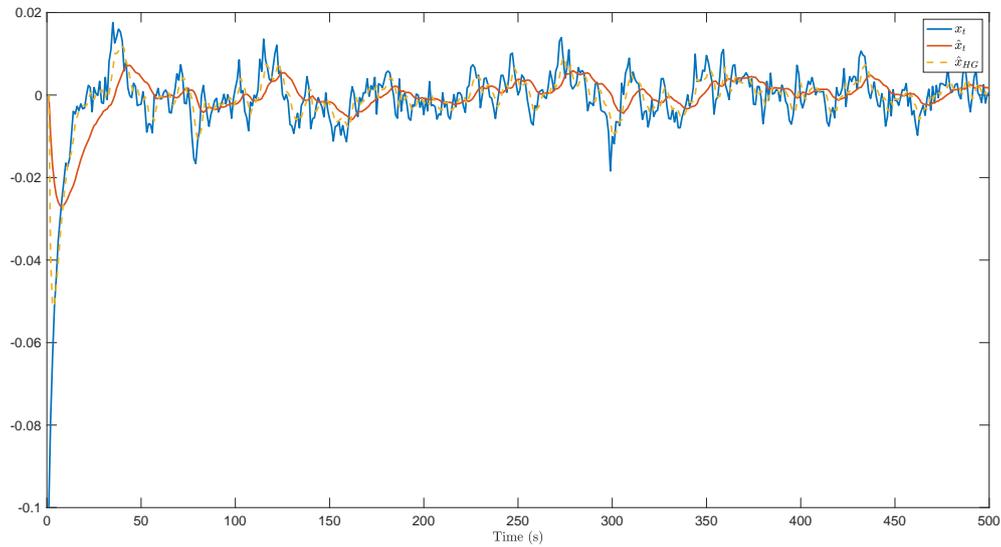
Figure 1. The estimation of the system state x_t.

the measured signal y_t. Using the proposed algorithm, the designed filter can be obtained as follows:

$$d\hat{x}_t = (-\hat{x}_t + \sin(\hat{x}_t) - v_t)\, dt,$$
$$dv_t = u_t dt. \tag{32}$$

Using the designed nonlinear filter, the estimation error e_t can be produced in a nonlinear form. Then, the nonlinear dynamics of this estimation error and the virtual signal z_t can be further formulated. Based on the presented backstepping design, the simulation results are developed in Figures 1-5, where the initial value of the compensative signal is 0 and the initial value of the system state is -0.1. Figure 1 demonstrates the estimation performance of the presented method where the measured signal and the estimated one are shown separately as a comparison. It is shown that the randomness in the measured signal is attenuated, while the estimated signal is close to the ideal signal without noise. A comparative study is also given in this figure using a high-gain observer. It is shown that the high-gain design achieves the state estimation with filtering effects. However, the performance is sensitive regarding to the gain value. Figure 2 indicates the estimation error signal. As in the aforementioned analysis, the estimation error would be described as a Brownian motion in ideal condition. However, in the computational simulation, the discretization has to be used where the trajectory is the increment of the Brownian motion. In particular, it is described as Gaussian white noise. Note that error $e_t rue$ is given in the figure where the value is given between the estimated value and the true value without noise. e_{HG} and e_t are the values between the measured value and the estimated values. e_{HG} implies that the high-gain design is closer to the measured value as an observer and the presented filtering algorithm leads to a result close to the true value without noise. Figures 3-5 illustrate the designed filtering compensative signal v_t, u_t, and the tracking error of virtual signal z_t. In addition, Figures 6 and 7 show the mean values of z_t and e_t. Notice that the mean values of both signals are bounded and close to zero. In addition, Figures 8 and 9 show the variance values of z_t and e_t, where the variance value of e_t is close to the assigned value 0.0001. Basically, in the assigned value there exists an error due to the influence of the filtering compensative signal. Based on these results, the design estimation objective for the investigated system is achieved.
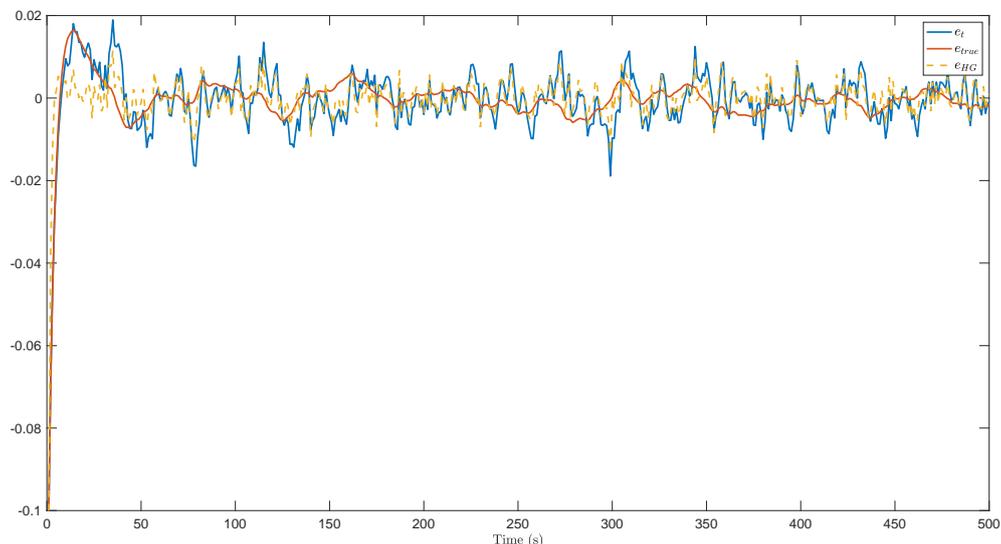
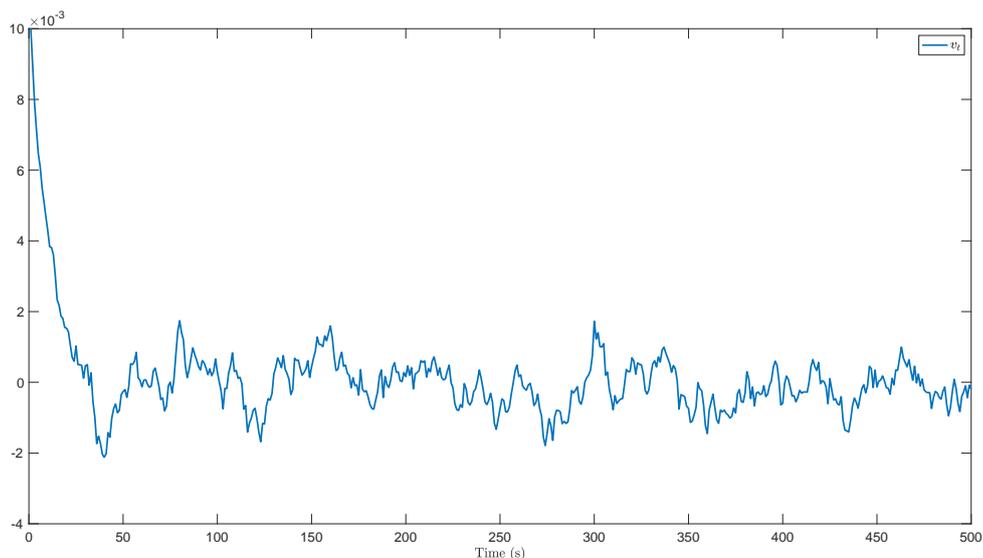**Figure 2.** The estimation error of the system state x_t.

Figure 3. The compensative signal v_t of filter with integrator.

6. DISCUSSION

Only the single variable estimation is considered above to simplify the backstepping design. The main challenge of extending the presented algorithm to the multivariate case is basically the multi-variable backstepping design. Inspired by multi-variable controller design [23], the block backstepping design is one of the potential solutions. The system model in Equation (1) can be extended to vector-valued form as follows:
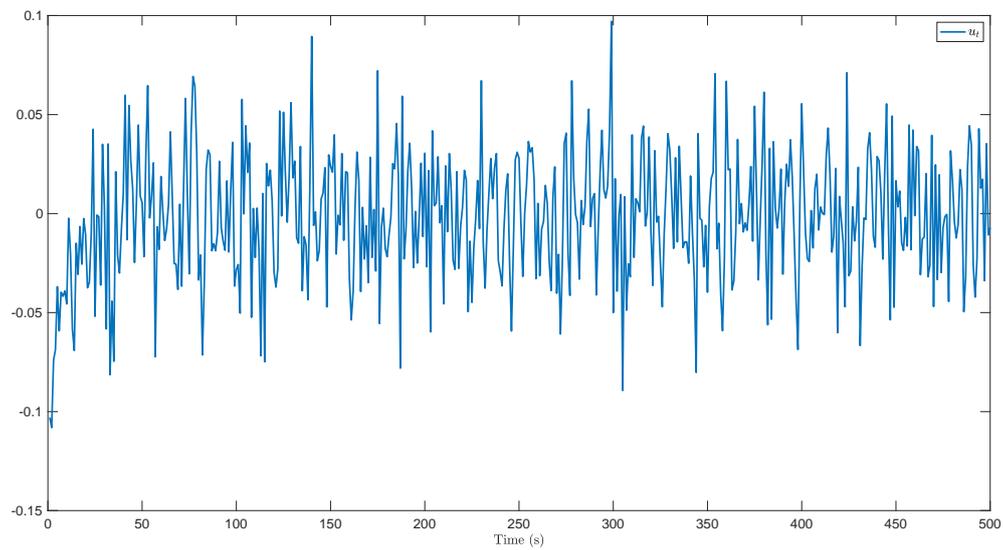
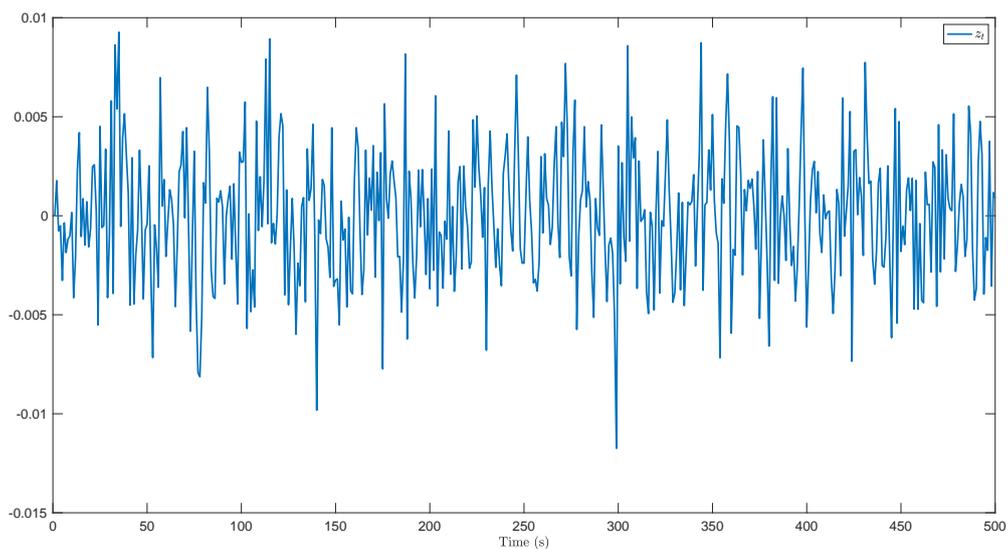**Figure 4.** The compensative signal u_t of filter before integral.

Figure 5. The trajectory of the virtual error z_t.

$$d\bar{x}_t = \bar{f}(\bar{x}_t)\, dt + \Sigma d\bar{W}_t,$$
$$y_t = Cx_t. \tag{33}$$

where $\bar{f}(\cdot)$ stands for a known smooth nonlinear function $\bar{f} : \mathbb{R}^{n \times 1} \to \mathbb{R}^n$, $\bar{W}_t$ denotes n-dimensional Wiener process, and Σ denotes a given as a real positive square matrix with n dimensions. $\bar{x}_t \in \mathbb{R}^n$ and $y_t \in \mathbb{R}^1$ stand for the system state vector and system output, respectively. $C \in \mathbb{R}^n$ denotes a vector-valued coefficient.
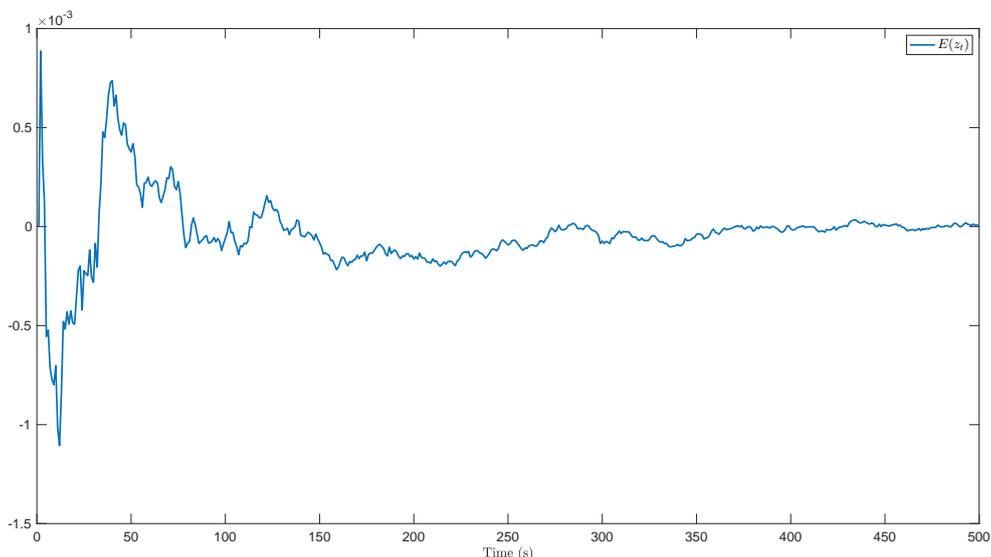
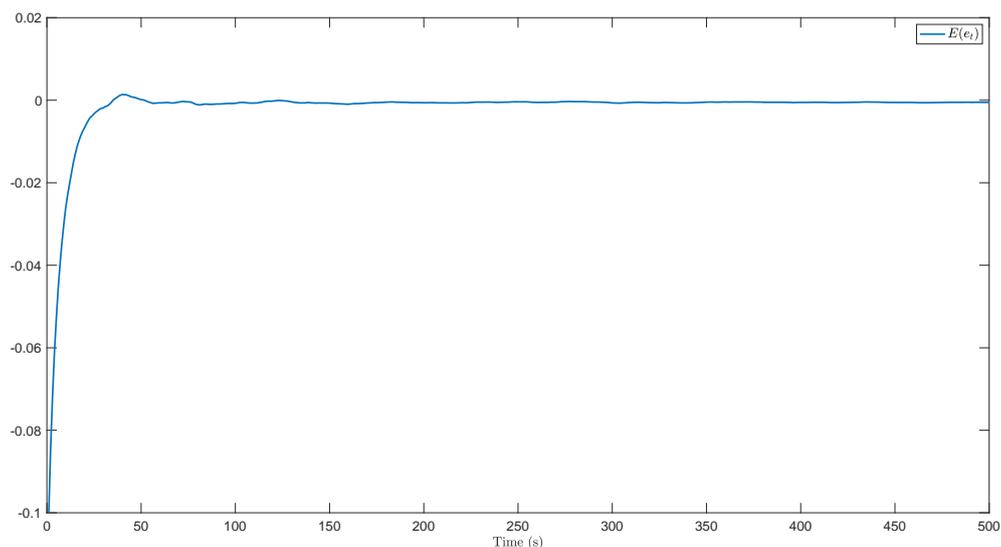**Figure 6.** The mean value of the virtual error signal z_t.

Figure 7. The mean value of the estimation error e_t.

Then, the filter structure can be confirmed in vectorized form.

$$d\hat{\bar{x}}_t = \left(\bar{f} \left(\hat{\bar{x}}_t \right) - \bar{v}_t \right) dt, \tag{34}$$

where $\hat{\bar{x}}_t$ denotes the estimated system state $\bar{x}_t$. $\bar{v}_t \in \mathbb{R}^n$ denotes the vector-based filtering compensative signal.

Similar to the design procedure, the Lyapunov functions can also be reused where the vector-value variables will be used. Since Lemma 2 holds for the multivariate system, the developed result in this paper can be extended directly following the block backstepping design. Notice that the linear Ornstein-Uhlenbeck process will be in the multi-dimensional form which leads to difficulty in solving the FPK equation as the joint
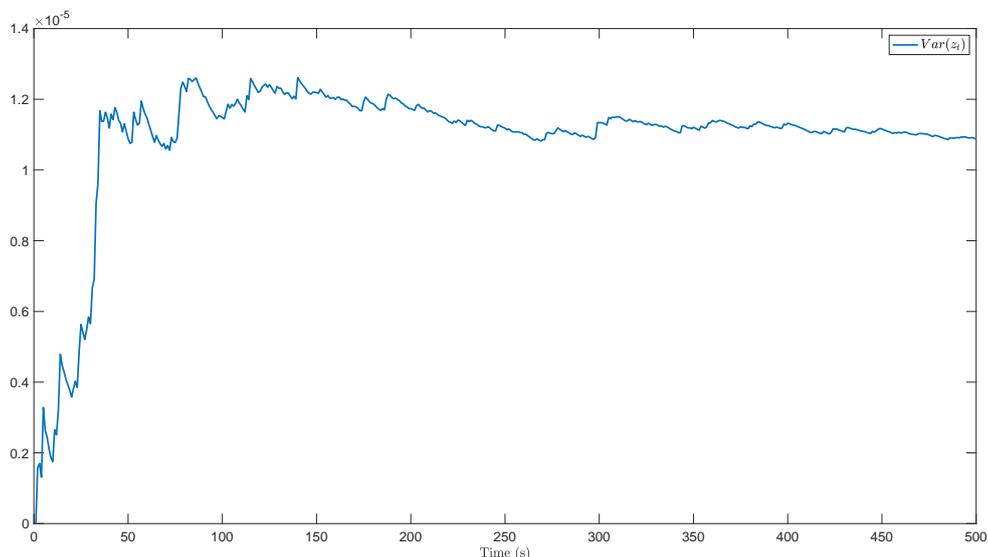
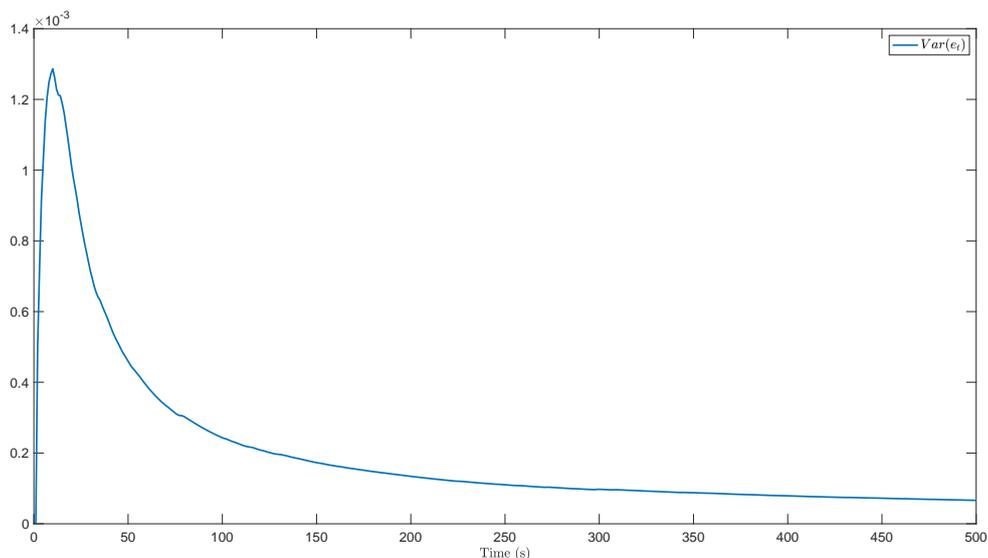**Figure 8.** The variance of the virtual error signal z_t.

Figure 9. The variance of the estimation error e_t.

probability density function has to be involved in the multivariate case. To avoid this problem, the design parameter θ should be selected as the positive diagonal matrix. Then, a set of FPK equations can be obtained where the vector state can be decomposed as multiple single variables. Therefore, the presented parameter selection scheme can also be reused for the multivariate system.

In addition, the measurement noise cannot be ignored in practice. An extended model should be formulated with the measurement noise which can be expressed as another stochastic differential equation. Thus, the convergence of the estimation error cannot be simply converted as the proposed linear dynamics. This extension will be considered as a future work with other assumptions.

7. CONCLUSION

The state estimation problem is investigated for a class of stochastic nonlinear systems, where the system model is described by the stochastic differential equation. To achieve the design objective, a new nonlinear filtering approach is designed. In particular, the design scheme is divided into two components: (1) The filtering structure can be confirmed based on the system model while the nonlinear estimation error can be further formulated. Then, an integrator is introduced into the estimation error for matching the backstepping design procedure. After that, the nonlinear dynamics can be converted to a linear Ornstein-Uhlenbeck process, where the mean value and variance value of the obtained estimation error is adjustable. (2) Since the variance value can be formulated analytically, the parameter can be designed for the filter. Ideally, the Brownian motion can be considered to obtain the desired variance value. Then, the design parameter in backstepping can be confirmed in order to attenuate the randomness of the state estimation. The simulation results and the multivariate system extension are also given to show the effectiveness and the potential extension of the presented estimation method. To extend the presented results, some comparative studies will be produced as future work where high-order sliding mode design [24,25] and adaptive high-gain design [26] will be further considered.

DECLARATIONS

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Authors' contributions
Made substantial contributions to conception and design of the study and performed data analysis and interpretation: Zhang Q
Performed data acquisition, as well as provided administrative, technical, and material support: Yin X

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Ethical approval and consent to participate
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REFERENCES

1. Barfoot TD. State estimation for robotics. Cambridge University Press; 2017.
2. Shukla N, Tiwari MK, Beydoun G. Next generation smart manufacturing and service systems using big data analytics. Elsevier; 2019.
3. Ben-Akiva M, Bierlaire M, Burton D, Koutsopoulos HN, Mishalani R. Network state estimation and prediction for real-time transportation management applications. In: Transportation Research Board 81st Annual Meeting; 2002. .
4. Tang X, Zhang Q, Hu L. An EKF-based performance enhancement scheme for stochastic nonlinear systems by dynamic set-point adjustment. *IEEE Access* 2020;8:62261–72.

5. Zhou Y, Zhang Q, Wang H, Zhou P, Chai T. Ekf-based enhanced performance controller design for nonlinear stochastic systems. *IEEE Transactions on Automatic Control* 2017;63:1155–62.

6. Kalman RE. A new approach to linear filtering and prediction problems 1960.

7. Ribeiro MI. Kalman and extended kalman filters: Concept, derivation and properties. *Institute for Systems and Robotics* 2004;43:46.

8. Xiong K, Zhang H, Chan C. Performance evaluation of UKF-based nonlinear filtering. *Automatica* 2006;42:261–70.

9. Kalman RE, Bucy RS. New results in linear filtering and prediction theory 1961.

10. Xiao X, Xi H, Zhu J, Ji H. Robust Kalman filter of continuous-time Markov jump linear systems based on state estimation performance. *International Journal of Systems Science* 2008;39:9–16.

11. Sarkka S. On unscented Kalman filtering for state estimation of continuous-time nonlinear systems. *IEEE Transactions on Automatic Control* 2007;52:1631–41.

12. Djuric PM, Kotecha JH, Zhang J, Huang Y, Ghirmai T, et al. Particle filtering. *IEEE signal processing magazine* 2003;20:19–38.

13. Guo L, Wang H. Minimum entropy filtering for multivariate stochastic systems with non-Gaussian noises. *IEEE Transactions on Automatic control* 2006;51:695–700.

14. Zhang Q. Performance enhanced Kalman filter design for non-Gaussian stochastic systems with data-based minimum entropy optimisation. *AIMS Electronics and Electrical Engineering* 2019;3:382–96.

15. Ghoreyshi A, Sanger TD. A nonlinear stochastic filter for continuous-time state estimation. *IEEE Transactions on Automatic Control* 2015;60:2161–65.

16. Ross SM, Kelly JJ, Sullivan RJ, Perry WJ, Mercer D, et al. Stochastic processes. vol. 2. Wiley New York; 1996.

17. Zhang Q, Wang Z, Wang H. Parametric covariance assignment using a reduced-order closed-form covariance model. *Systems Science & Control Engineering* 2016;4:78–86.

18. Zhang Q, Wang H. A Novel Data-based Stochastic Distribution Control for Non-Gaussian Stochastic Systems. *IEEE Transactions on Automatic Control* 2021.

19. Zhang Q, Sepulveda F. A model study of the neural interaction via mutual coupling factor identification. In: 2017 39th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC). IEEE; 2017. pp. 3329–32.

20. Zhang Q, Sepulveda F. A statistical description of pairwise interaction between nerve fibres. In: 2017 8th International IEEE/EMBS Conference on Neural Engineering (NER). IEEE; 2017. pp. 194–98.

21. Minskii R. Stochastic stability of differential equations, vol. 7 of Monographs and Textbooks on Mechanics of Solids and Fluids: Mechanics and Analysis. Sijthoff & Noordhoff, Alphen aan den Rijn 1980.

22. Liu SJ, Zhang JF, Jiang ZP. Decentralized adaptive output-feedback stabilization for large-scale stochastic nonlinear systems. *Automatica* 2007;43:238–51.

23. Zhang QC, Hu L, Gow J. Output feedback stabilization for MIMO semi-linear stochastic systems with transient optimisation. *International Journal of Automation and Computing* 2020;17:83–95.

24. Bejarano FJ, Fridman L. High order sliding mode observer for linear systems with unbounded unknown inputs. *International Journal of Control* 2010;83:1920–29.

25. Benallegue A, Mokhtari A, Fridman L. High-order sliding-mode observer for a quadrotor UAV. *International Journal of Robust and Nonlinear Control: IFAC-Affiliated Journal* 2008;18:427–40.

26. Boizot N, Busvelle E, Gauthier JP. An adaptive high-gain observer for nonlinear systems. *Automatica* 2010;46:1483–88.