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Supplementary Materials

A decision-making model for echelon utilization of retired batteries in competitive duopoly: the role of government subsidy

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Proof of Proposition 1: We first calculate the second-order partial derivatives of π_1 and π_2 to p_1 and p_2, respectively. Since $\frac{\partial^2\pi_1}{\partial {p_1}^2}=\frac{\partial^2\pi_2}{\partial {p_2}^2}=-2\beta<0$, π_1 and π_2 are both concave functions of p_1 and p_2. Let the first-order partial derivatives of π_1 and π_2 to p_1 and p_2 be zero, so we can get ${p_1}^*$ and ${p_2}^*$ and substitute them into π_r. Since $\frac{\partial^2\pi_r}{\partial w^2}=\frac{4\beta(-\beta+\gamma)}{2\beta-\gamma}$ and Assumption 4, we know that $\frac{\partial^2\pi_r}{\partial w^2}<0$ and π_r is a concave function of w. Let the first-order partial derivative of π_r to w be zero, and we can get w^* and then substitute them into π_m. If we compute the second-order partial derivatives of π_m to P and F, respectively, then we can get the Hessian matrix as $\begin{bmatrix}\frac{\partial^2\pi_m}{\partial P^2} & \frac{\partial^2\pi_m}{\partial P\partial F}\\ \frac{\partial^2\pi_m}{\partial F\partial P} & \frac{\partial^2\pi_m}{\partial F^2}\end{bmatrix}=\begin{bmatrix}-2b & 0\\ 0 & \frac{2\beta(\gamma-\beta)}{2\beta-\gamma}\end{bmatrix}$. According to Assumption 4, we know that the matrix is negative definite. Let the first-order partial derivatives of π_m to P and F be zero, then we can get P^* and F^* and substitute them into previous results. We get the final ${p_1}^*$, ${p_2}^*$, w^*, ${T_1}^*$, ${T_2}^*$, Q^*, G^*, ${\pi_m}^*$, ${\pi_r}^*$, ${\pi_1}^*$, ${\pi_2}^*$.

Proof of Corollary 1: Let the first-order partial derivatives of ${p_1}^*$ and ${p_2}^*$ to C_1 and C_2, then we can get $\frac{\partial {p_1}^*}{\partial C_1}=\frac{\beta(10\beta-3\gamma)}{32\beta^2-8\gamma^2}, \frac{\partial {p_1}^*}{\partial C_2}=\frac{\beta(-6\beta+5\gamma)}{32\beta^2-8\gamma^2}, \frac{\partial {p_2}^*}{\partial C_1}=\frac{\beta(-6\beta+5\gamma)}{32\beta^2-8\gamma^2}, \frac{\partial {p_2}^*}{\partial C_2}=\frac{\beta(10\beta-3\gamma)}{32\beta^2-8\gamma^2}$, which can be proven by discussing the positivity and negativity of four partial derivatives according to Assumption 4.

Given the first-order partial derivatives of w^* to C_1 and C_2, respectively, then we can get $\frac{\partial w^*}{\partial C_1}=\frac{\partial w^*}{\partial C_2}=-\frac{1}{8}$.

Proof of Corollary 2: The four conclusions in Corollary 2 have similar proofs, so for simplicity, we prove only conclusion (1) here. Because $\frac{\partial^2 {\pi_1}^*}{\partial {C_1}^2}=\frac{\beta(-10\beta^2-3\beta\gamma+5\gamma^2)^2}{32(-4\beta^2+\gamma^2)^2}>0$, the first-order partial derivative of ${\pi_1}^*$ to C_1 is monotonically increasing. Let $\frac{\partial {\pi_1}^*}{\partial C_1}=0$, then we can get $C_1=\frac{1}{10\beta^2+3\beta\gamma-5\gamma^2}((4\beta^2-2\beta\gamma-2\gamma^2)C_0+(6\beta^2+5\beta\gamma-3\gamma^2)C_2+2(2\beta+\gamma)(e-h\beta+h\gamma+(\beta-\gamma)C_m+(-\beta+\gamma)C_r))$.

According to monotonicity, (1) can be proved.

Proof of Corollary 3: We can easily get the first-order partial derivative of $p_1{}^{s*}$ to S as $\frac{\partial p_1{}^{s*}}{\partial S} = \frac{\beta(-10\beta+3\gamma)}{32\beta^2-8\gamma^2}$. According to Assumption 4, it is easy to know that $4\beta^2 - \gamma^2 > 0$ and $3\gamma - 10\beta < 0$. From this, we can prove that $\frac{\partial p_1{}^{s*}}{\partial S} < 0$ and show similarly that $\frac{\partial p_2{}^{s*}}{\partial S} > 0$. We can easily get the first-order partial derivative of w^{s*} to S as $\frac{\partial w^{s*}}{\partial S} = \frac{1}{8} > 0$ and, thus, prove Corollary 3.

Proof of Corollary 4: The two conclusions in Corollary 4 are proved similarly, so we prove just (1) here. Because $\frac{\partial^2 \pi_1{}^{s*}}{\partial S^2} = \frac{\beta(10\beta^2+3\beta\gamma-5\gamma^2)^2}{32(-4\beta^2+\gamma^2)^2} > 0$, the first-order partial derivative of $\pi_1{}^{s*}$ to S is monotonically increasing. Let $\frac{\partial \pi_1{}^{s*}}{\partial S} = 0$, then we can get

$$S = \frac{1}{10\beta^2+3\beta\gamma-5\gamma^2}(-4e\beta + 4h\beta^2 - 2e\gamma - 2h\beta\gamma - 2h\gamma^2 + 2(-2\beta^2 + \beta\gamma + \gamma^2)C_0 + (10\beta^2 + 3\beta\gamma - 5\gamma^2)C_1 - 6\beta^2 C_2 - 5\beta\gamma C_2 + 3\gamma^2 C_2 - 4\beta^2 C_m + 2\beta\gamma C_m + 2\gamma^2 C_m + 4\beta^2 C_r - 2\beta\gamma C_r - 2\gamma^2 C_r).$$

(1) can, thus, be proved according to the principles of monotonicity.

Table 1. The profits of each member of the supply chain without subsidies

$T_1{}^* = \frac{1}{32\beta^2 - 8\gamma^2}\beta(4e\beta - 4h\beta^2 + 2e\gamma + 2h\beta\gamma + 2h\gamma^2 + (4\beta^2 - 2\beta\gamma - 2\gamma^2)C_0 + (-10\beta^2 - 3\beta\gamma + 5\gamma^2)C_1 + 6\beta^2 C_2 + 5\beta\gamma C_2 - 3\gamma^2 C_2 + 4\beta^2 C_m - 2\beta\gamma C_m - 2\gamma^2 C_m - 4\beta^2 C_r + 2\beta\gamma C_r + 2\gamma^2 C_r$
$T_2{}^* = \frac{1}{32\beta^2 - 8\gamma^2}\beta(4e\beta - 4h\beta^2 + 2e\gamma + 2h\beta\gamma + 2h\gamma^2 + (4\beta^2 - 2\beta\gamma - 2\gamma^2)C_0 + (6\beta^2 + 5\beta\gamma - 3\gamma^2)C_1 - 10\beta^2 C_2 - 3\beta\gamma C_2 + 5\gamma^2 C_2 + 4\beta^2 C_m - 2\beta\gamma C_m - 2\gamma^2 C_m - 4\beta^2 C_r + 2\beta\gamma C_r + 2\gamma^2 C_r)$ $\pi_m{}^* = \frac{1}{16b(\beta - \gamma)(2\beta - \gamma)}(4be^2\beta + 8a^2\beta^2 - 8beh\beta^2 + 4bh^2\beta^3 - 12a^2\beta\gamma + 8beh\beta\gamma - 8bh^2\beta^2\gamma + 4a^2\gamma^2 + 4bh^2\beta\gamma^2 + 4b\beta(\beta - \gamma)^2 C_0^2 + b\beta(\beta - \gamma)^2 C_1^2 - 4be\beta^2 C_2 + 4bh\beta^3 C_2 + 4be\beta\gamma C_2 - 8bh\beta^2\gamma C_2 + 4bh\beta\gamma^2 C_2 + b\beta^3 C_2^2 - 2b\beta^2\gamma C_2^2 + b\beta\gamma^2 C_2^2 - 16ab\beta^2 C_m + 8be\beta^2 C_m - 8bh\beta^3 C_m + 24ab\beta\gamma C_m - 8be\beta\gamma C_m + 16bh\beta^2\gamma C_m - 8ab\gamma^2 C_m - 8bh\beta\gamma^2 C_m - 4b\beta^3 C_2 C_m + 8b\beta^2\gamma C_2 C_m - 4b\beta\gamma^2 C_2 C_m + 8b^2\beta^2 C_m^2 + 4b\beta^3 C_m^2 - 12b^2\beta\gamma C_m^2 - 8b\beta^2\gamma C_m^2 + 4b^2\gamma^2 C_m^2 + 4b\beta\gamma^2 C_m^2 - 8be\beta^2 C_r + 8bh\beta^3 C_r + 8be\beta\gamma C_r - 16bh\beta^2\gamma C_r + 8bh\beta\gamma^2 C_r + 4b\beta^3 C_2 C_r - 8b\beta^2\gamma C_2 C_r + 4b\beta\gamma^2 C_2 C_r - 8b\beta^3 C_m C_r + 16b\beta^2\gamma C_m C_r - 8b\beta\gamma^2 C_m C_r + 4b\beta^3 C_r^2 - 8b\beta^2\gamma C_r^2 + 4b\beta\gamma^2 C_r^2 + 2b\beta(\beta - \gamma)C_1((\beta - \gamma)C_2 - 2(e - h\beta + h\gamma + (\beta - \gamma)C_m + (-\beta + \gamma)C_r)) - 4b\beta(\beta - \gamma)C_0((\beta - \gamma)C_1 + (\beta - \gamma)C_2 - 2(e - h\beta + h\gamma + (\beta - \gamma)C_m + (-\beta + \gamma)C_r)))$
$\pi_r{}^* = \frac{1}{32(2\beta^2 - 3\beta\gamma + \gamma^2)}(4\beta(\beta - \gamma)^2 C_0^2 + \beta((-\beta + \gamma)C_1 + (-\beta + \gamma)C_2 + 2(e - h\beta + h\gamma + (\beta - \gamma)C_m + (-\beta + \gamma)C_r))^2 + 4(\beta - \gamma)C_0(\beta(-\beta + \gamma)C_1 + \beta(-\beta + \gamma)C_2 + 2(e\beta - h\beta^2 + h\beta\gamma - 4a\beta\theta + 2a\gamma\theta + (\beta^2 - \beta\gamma + 4b\beta\theta - 2b\gamma\theta)C_m + \beta(-\beta + \gamma)C_r)))$
$\pi_1{}^* = \frac{1}{64(-4\beta^2 + \gamma^2)^2}\beta(4e\beta - 4h\beta^2 + 2e\gamma + 2h\beta\gamma + 2h\gamma^2 - 2(-2\beta^2 + \beta\gamma + \gamma^2)C_0 - (10\beta^2 + 3\beta\gamma - 5\gamma^2)C_1 + 6\beta^2 C_2 + 5\beta\gamma C_2 - 3\gamma^2 C_2 + 4\beta^2 C_m - 2\beta\gamma C_m - 2\gamma^2 C_m - 4\beta^2 C_r + 2\beta\gamma C_r + 2\gamma^2 C_r)^2$
$\pi_2{}^* = \frac{1}{64(-4\beta^2 + \gamma^2)^2}\beta(-4e\beta + 4h\beta^2 - 2e\gamma - 2h\beta\gamma - 2h\gamma^2 + 2(-2\beta^2 + \beta\gamma + \gamma^2)C_0 + (-6\beta^2 - 5\beta\gamma + 3\gamma^2)C_1 + 10\beta^2 C_2 + 3\beta\gamma C_2 - 5\gamma^2 C_2 - 4\beta^2 C_m + 2\beta\gamma C_m + 2\gamma^2 C_m + 4\beta^2 C_r - 2\beta\gamma C_r - 2\gamma^2 C_r)^2$

Table 2. The profits of each member of the supply chain with subsidies

${T_1}^{s*} = \frac{1}{32\beta^2 - 8\gamma^2}\beta(4e\beta - 4h\beta^2 + 10S\beta^2 + 2e\gamma + 2h\beta\gamma + 3S\beta\gamma + 2h\gamma^2 - 5S\gamma^2 + (4\beta^2 - 2\beta\gamma - 2\gamma^2)C_0 + (-10\beta^2 - 3\beta\gamma + 5\gamma^2)C_1 + 6\beta^2 C_2 + 5\beta\gamma C_2 - 3\gamma^2 C_2 + 4\beta^2 C_m - 2\beta\gamma C_m - 2\gamma^2 C_m - 4\beta^2 C_r + 2\beta\gamma C_r + 2\gamma^2 C_r)$
${T_2}^{s*} = \frac{1}{32\beta^2 - 8\gamma^2}\beta(4e\beta - 4h\beta^2 - 6S\beta^2 + 2e\gamma + 2h\beta\gamma - 5S\beta\gamma + 2h\gamma^2 + 3S\gamma^2 + (4\beta^2 - 2\beta\gamma - 2\gamma^2)C_0 + (6\beta^2 + 5\beta\gamma - 3\gamma^2)C_1 - 10\beta^2 C_2 - 3\beta\gamma C_2 + 5\gamma^2 C_2 + 4\beta^2 C_m - 2\beta\gamma C_m - 2\gamma^2 C_m - 4\beta^2 C_r + 2\beta\gamma C_r + 2\gamma^2 C_r)$
${\pi_m}^{s*} = \frac{1}{16b(\beta - \gamma)(2\beta - \gamma)}(4be^2\beta + 8a^2\beta^2 - 8beh\beta^2 + 4beS\beta^2 + 4bh^2\beta^3 - 4bhS\beta^3 + bS^2\beta^3 - 12a^2\beta\gamma + 8beh\beta\gamma - 4beS\beta\gamma - 8bh^2\beta^2\gamma + 8bhS\beta^2\gamma - 2bS^2\beta^2\gamma + 4a^2\gamma^2 + 4bh^2\beta\gamma^2 - 4bhS\beta\gamma^2 + bS^2\beta\gamma^2 + 4b\beta(\beta - \gamma)^2 C_0^2 + b\beta(\beta - \gamma)^2 C_1^2 - 4be\beta^2 C_2 + 4bh\beta^3 C_2 - 2bS\beta^3 C_2 + 4be\beta\gamma C_2 - 8bh\beta^2\gamma C_2 + 4bS\beta^2\gamma C_2 + 4bh\beta\gamma^2 C_2 - 2bS\beta\gamma^2 C_2 + b\beta^3 C_2^2 - 2b\beta^2\gamma C_2^2 + b\beta\gamma^2 C_2^2 - 16ab\beta^2 C_m + 8be\beta^2 C_m - 8bh\beta^3 C_m + 4bS\beta^3 C_m + 24ab\beta\gamma C_m - 8be\beta\gamma C_m + 16bh\beta^2\gamma C_m - 8bS\beta^2\gamma C_m - 8ab\gamma^2 C_m - 8bh\beta\gamma^2 C_m + 4bS\beta\gamma^2 C_m - 4b\beta^3 C_2 C_m + 8b\beta^2\gamma C_2 C_m - 4b\beta\gamma^2 C_2 C_m + 8b^2\beta^2 C_m^2 + 4b\beta^3 C_m^2 - 12b^2\beta\gamma C_m^2 - 8b\beta^2\gamma C_m^2 + 4b^2\gamma^2 C_m^2 + 4b\beta\gamma^2 C_m^2 - 8be\beta^2 C_r + 8bh\beta^3 C_r - 4bS\beta^3 C_r + 8be\beta\gamma C_r - 16bh\beta^2\gamma C_r + 8bS\beta^2\gamma C_r + 8bh\beta\gamma^2 C_r - 4bS\beta\gamma^2 C_r + 4b\beta^3 C_2 C_r - 8b\beta^2\gamma C_2 C_r + 4b\beta\gamma^2 C_2 C_r - 8b\beta^3 C_m C_r + 16b\beta^2\gamma C_m C_r - 8b\beta\gamma^2 C_m C_r + 4b\beta^3 C_r^2 - 8b\beta^2\gamma C_r^2 + 4b\beta\gamma^2 C_r^2 + 2b\beta(\beta - \gamma)C_1(-2e + 2h\beta - S\beta - 2h\gamma + S\gamma + (\beta - \gamma)C_2 - 2(\beta - \gamma)C_m + 2\beta C_r - 2\gamma C_r) - 4b\beta(\beta - \gamma)C_0(-2e + 2h\beta - S\beta - 2h\gamma + S\gamma + (\beta - \gamma)C_1 + (\beta - \gamma)C_2 - 2\beta C_m + 2\gamma C_m + 2\beta C_r - 2\gamma C_r))$
${\pi_r}^{s*} = \frac{1}{32(2\beta^2 - 3\beta\gamma + \gamma^2)}(4\beta(\beta - \gamma)^2 C_0^2 + \beta(2e - 2h\beta + S\beta + 2h\gamma - S\gamma + (-\beta + \gamma)C_1 + (-\beta + \gamma)C_2 + 2\beta C_m - 2\gamma C_m - 2\beta C_r + 2\gamma C_r)^2 + 4(\beta - \gamma)C_0(2e\beta - 2h\beta^2 + S\beta^2 + 2h\beta\gamma - S\beta\gamma - 8a\beta\theta + 4a\gamma\theta + \beta(-\beta + \gamma)C_1 + \beta(-\beta + \gamma)C_2 + 2\beta^2 C_m - 2\beta\gamma C_m + 8b\beta\theta C_m - 4b\gamma\theta C_m - 2\beta^2 C_r + 2\beta\gamma C_r))$
${\pi_1}^{s*} = \frac{1}{64(-4\beta^2 + \gamma^2)^2}\beta(4e\beta - 4h\beta^2 + 10S\beta^2 + 2e\gamma + 2h\beta\gamma + 3S\beta\gamma + 2h\gamma^2 - 5S\gamma^2 - 2(-2\beta^2 + \beta\gamma + \gamma^2)C_0 - (10\beta^2 + 3\beta\gamma - 5\gamma^2)C_1 + 6\beta^2 C_2 + 5\beta\gamma C_2 - 3\gamma^2 C_2 + 4\beta^2 C_m - 2\beta\gamma C_m - 2\gamma^2 C_m - 4\beta^2 C_r + 2\beta\gamma C_r + 2\gamma^2 C_r)^2$
${\pi_2}^{s*} = \frac{1}{64(-4\beta^2 + \gamma^2)^2}\beta(-4e\beta + 4h\beta^2 + 6S\beta^2 - 2e\gamma - 2h\beta\gamma + 5S\beta\gamma - 2h\gamma^2 - 3S\gamma^2 + 2(-2\beta^2 + \beta\gamma + \gamma^2)C_0 + (-6\beta^2 - 5\beta\gamma + 3\gamma^2)C_1 + 10\beta^2 C_2 + 3\beta\gamma C_2 - 5\gamma^2 C_2 - 4\beta^2 C_m + 2\beta\gamma C_m + 2\gamma^2 C_m + 4\beta^2 C_r - 2\beta\gamma C_r - 2\gamma^2 C_r)^2$